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Almost-periodic solutions of a delay population equation with feedback control☆

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Abstract

By means of the properties of an almost-periodic system and the Lyapunov–Razumikhin technique, sufficient conditions are obtained for the existence of almost-periodic solution of delay population equation with feedback control.

MSC: 34K; 34D05; 34D08

Keywords: Almost-periodic solution; Feedback control; Population equation; Lyapunov–Razumikhin methods

1. Introduction

Seifert [9] have studied the delay differential equation for single species population variation:

$$\dot{x}(t) = x(t)[a - bx(t) - x(t-1)]. \tag{1}$$

He proved that if $b > 1$, all positive solutions converge to $a/(b+1)$ which is a solution of Eq. (1), that is, $x(t) = a/(b+1)$ is a global attractor. In 1996, Yuan [15] consider the

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nonautonomous case of system (1), i.e.,

$$\dot{x}(t) = x(t)[a(t) - b(t)x(t) - x(t-1)], \tag{2}$$

where $a(t) > 0$, $b(t) > 0$ are almost periodic. Eq. (2) may be regarded as a prototypical model for the variation of the population of an organism, when there is density-dependent growth which depends not only on the population at time t, but also on the population one time-unit earlier. Let $a^L = \min_{t\in R} a(t)$, $a^M = \sup_{t\in R} a(t)$, $b^L = \min_{t\in R} b(t)$, $b^M = \sup_{t\in R} b(t)$, Yuan proved that if

$$a^L > \frac{a^M/a^L}{(a^L - a^M/a^L)/b^M}$$

holds, system (2) admits a strictly positive almost-periodic solution $q(t)$, which is uniformly asymptotically stable and $\mathrm{mod}(q(t)) \subset \mathrm{mod}(a(t), b(t))$. This result is a generalization of Seifert's result.

On the other hand, in some situations, one may wish to alter the position of $x(t)$, but to keep its stability. This is of significance in the control procedure of ecology balance. One of the techniques to achieve this aim is to alter system (2) structurally by introducing "indirect control" variables. Therefore we consider the following delay population equation with feedback control:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)[r(t) - a(t)x(t) - x(t-\tau) - c(t)u(t)], \\ \dfrac{\mathrm{d}u(t)}{\mathrm{d}t} = -\eta(t)u(t) + g(t)x(t-\tau), \end{cases} \tag{3}$$

where $r(t), a(t), c(t), \eta(t), g(t)$ are continuous, positive almost-periodic functions.

Though much work has been done for logistic model and related feedback control system [1–4,6,7,10–13], most of the works dealt with positive periodic solution of the system considered, few scholars investigated the almost-periodic solution of delay population equation with feedback control. In this paper, we use the properties of almost-periodic system and Lyapunov–Razumikhin technique to obtain the existence of almost-periodic solution of system (3). The theory of almost-periodic function can be found in [5,8,14].

Definition 1 (*Fink [5]*). A function $f(t)$ is said to be almost periodic, if for any $\varepsilon > 0$, there is a constant $l(\varepsilon) > 0$, such that in any interval of length $l(\varepsilon)$ there exists τ such that the inequality

$$|f(t+\tau) - f(t)| < \varepsilon$$

is satisfied for all $t \in (-\infty, +\infty)$. The number τ is called an ε-translation number of $f(t)$.

Let $\{\lambda_j\}$ denote the set of all real numbers such that

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T f(t)\exp(-i\lambda_j t)\,\mathrm{d}t \neq 0. \tag{4}$$

It is well known that the set of numbers $\{\lambda_j\}$ in (4) is countable. The set $\{\sum_1^N n_j\lambda_j\}$ for all integers N and integers n_j is called the module of $f(t)$, denoted by $\mathrm{mod}(f)$.