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Existence and global exponential stability of periodic solution for Cohen–Grossberg neural networks with delays☆

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Abstract

This paper is concerned with the existence and global exponential stability of periodic solutions for a nonlinear periodic system, arising from the description of the states of neurons in delayed Cohen–Grossberg type. By the continuation theorem of coincidence degree theory and Lyapunov functionals technique, we deduce some sufficient conditions ensuring existence as well as global exponential stability of periodic solution. Some existing results are improved and extended. Even corresponding to an autonomous system, our results that these conditions are milder and less restrictive than previous known criteria since the hypothesis of boundedness and differentiability on the activation function are dropped. The theoretical analyses are verified by numerical simulations.

MSC: 34C25; 34D23

Keywords: Existence; Global exponential stability; Periodic solution; Degree theory; Neural networks

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1. Introduction

Neural networks can be represented as differential equations that describe the evolution of the model as functions of time. These differential equations have received increasing interest due to their promising potential applications in areas such as classification, parallel computing, associative memory and solving some optimization problem. Such applications heavily depend on the dynamic behavior of networks, therefore, the analysis of these dynamic behaviors is a necessary step for practical design of neural networks. However, time delays are unavoidably encountered in the implementation of neural networks. For example, delay occurs due to the finite speeds of the switching and transmission of signals in a network. This leads to the delayed neural networks that were first explicitly introduced in [17]. Since then, the delayed neural networks have been widely studied and some progress has been made.

In this paper, we consider the general Cohen–Grossberg type neural network with delays described by the following periodic system:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -a_i(x_i(t))\left[b_i(t,x_i(t)) - \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t)) - \sum_{j=1}^{n} b_{ij}(t) f_j(x_j(t-\tau_{ij})) + u_i(t)\right], \quad t \geqslant 0, \quad i = 1,2,\ldots,n, \tag{1.1}$$

where $n \geqslant 2$ is the number of neurons in the network, $x_i(t)$ denotes the state variable associated with the ith neuron, the activation function $f_j(x_j(t))$ denotes the output of the jth neuron at time t, the continuous functions $a_{ij}(t)$, $b_{ij}(t)$, $u_i(t)$ are ω-periodic: $a_{ij}(t)$ weights the strength of the jth neuron on the ith neuron at time t; $b_{ij}(t)$ the strength of the jth neuron on the ith neuron at time $t-\tau_{ij}$; $u_i(t)$ denotes the input to ith neuron; τ_{ij} corresponds to the transmission delay along the axon of jth neuron and is a nonnegative constant, the positive continuous function $a_i\colon R \to R^+$ represents an amplification function and there exist two positive constant numbers $\underline{\alpha}_i$ and $\overline{\alpha}_i$ such that

$$0 < \underline{\alpha}_i \leqslant a_i(v) \leqslant \overline{\alpha}_i, \quad \forall v \in R, \quad i = 1,2,\ldots,n, \tag{1.2}$$

the continuous function $b_i\colon R^+ \times R \to R$ denotes an appropriately behaved function which is ω-periodic with respect to its first argument and there exists some continuous positive ω-periodic function $c_i(t)$ such that

$$\frac{b_i(t,u) - b_i(t,v)}{u-v} \geqslant c_i(t) > 0, \quad \forall u \neq v, \quad u, v \in R, \quad i = 1,2,\ldots,n. \tag{1.3}$$

The initial conditions associated with (1.1) are given in form

$$x_i(s) = \phi_i(s) \in C([-\tau,0],R), \quad i = 1,2,\ldots,n, \tag{1.4}$$

where $\tau = \max_{1\leqslant i,j\leqslant n}\{\tau_{ij}\}$.

System (1.1) is quite general and it includes several well-known neural networks models as its special cases such as Hopfield neural networks [1,7–9,11,20,21,24,25,28,29], cellular neural networks [4,5,15,18] etc. For the case of the function $b_i(t,v)$ is only a function with respect to its second argument, the coefficients $a_{ij}(t)$, $b_{ij}(t)$ and the input function $u_i(t)$