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ScienceDirect
Nonlinear Analysis: Real World Applications 9 (2008) 1361 – 1377
Nonlinear Analysis
Real World Applications
www.elsevier.com/locate/na

Dynamics of a small vibro-impact pile driver

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Received 18 February 2006; accepted 6 March 2007

Abstract

A mathematical model is developed to study periodic-impact motions and bifurcations in dynamics of a small vibro-impact pile driver. Dynamics of the small vibro-impact pile driver can be analyzed by means of a three-dimensional map, which describes free flight and sticking solutions of the vibro-impact system, between impacts, supplemented by transition conditions at the instants of impacts. Piecewise property and singularity are found to exist in the Poincaré map. The piecewise property is caused by the transitions of free flight and sticking motions of the driver and the pile immediately after the impact, and the singularity of map is generated via the grazing contact of the driver and the pile immediately before the impact. These properties of the map have been shown to exhibit particular types of sliding and grazing bifurcations of periodic-impact motions under parameter variation. The influence of piecewise property, grazing singularities and parameter variation on the performance of the vibro-impact pile driver is analyzed. The global bifurcation diagrams for the impact velocity of the driver versus the forcing frequency are plotted to predict much of the qualitative behavior of the actual physical system, which enable the practicing engineer to select excitation frequency ranges in which stable period one single-impact response can be expected to occur, and to predict the larger impact velocity and shorter impact period of such response.

Keywords: Impact; Vibration; Periodic motion; Bifurcation; Chaos

1. Introduction

Vibratory pile driver generally makes the pile sink into the soil by means of its dead weight. With decrease in the mass of the pile driver, piling efficiency becomes very low, and it is difficult to sink the pile into the harder soil by its dead weight. So it is not easily brought into effect to reduce the pile drivers in size. However, small pile drivers have also a vast range of prospects of application in the piling engineering. When controlling flood and rushing to deal with an emergency on the water conservancy building sites, it is urgent for the small pile drivers to substitute for manual piling to raise working efficiency. However, this problem may be virtually resolved by introducing the vibro-impact pile drivers. The driver impacts against the pile with very high speed in the vibrating process, and the impact causes the pile to sink into the soil more effectively. Dynamics of vibratory pile drivers is generally analyzed by using vibration theory of one-degree-of-freedom oscillator [44]. However, the research into non-linear dynamics of the vibro-impact pile driver needs one to consider simultaneously both vibration and impact. Because the optimization of system parameters of the vibro-impact pile driver is very difficult due to the different soil and working conditions, its progress in theory

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1468-1218/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.nonrwa.2007.03.004

and practice is slower than that of vibratory pile driver. So the study of dynamics of small vibro-impact pile drivers is of a considerable importance in optimum design and working efficiency of such machines. The study will bring a great advance in developing and improving on the products of small vibro-impact pile drivers, and give a fresh impetus to the development of large pile drivers.

The small pile driver is a typical vibro-impact machine. Vibro-impact systems exist in a wide variety of engineering applications, particularly in mechanisms and machines with clearances or rigid stops. The principles of operation of vibration hammers, impact dampers, inertial shakers, pile drivers, milling and forming machines, etc., are based on the impact action for moving bodies. With other equipment, e.g., machines with clearances, heat exchangers, steam generator tubes, fuel rods in nuclear power plants, rolling railway wheelset, gear transmissions and so on, impacts also occur, but they are undesirable as they bring about failures, strain, shorter service life and increased noise levels. The dynamics of vibro-impact systems is of a considerable importance in noise suppression, reliability analysis and optimum design of machines with clearances or rigid obstacles. The trajectories of such systems in phase space have discontinuities caused by the impacts. The presence of the non-linearity and discontinuity complicates the dynamic analysis of such systems considerably, but they can be described theoretically and numerically by discontinuities in good agreement with reality. Compared with single impact, the non-linear dynamics of vibro-impact systems is more complicated, and hence, has received great attention. The large interest in analyzing and understanding the performance of such systems is reflected by vast and ever increasing amount of research effort devoted in this area. Some important problems on vibro-impact dynamics, such as stability and global bifurcations [1,5,19,23,33,39,45,49], grazing singularity of impact mapping [3,7,9,10,15,16,24,28,34,37,42,43,55], chattering impacts [36], intermittency chaos [41], sliding bifurcation [46,47,52,53], quasi-periodic impacts [4,25,26,29,31,35], controlling chaos [13,14,22] and experimental study [54], etc., have been studied in the past several years. Along with the basic research into vibro-impact dynamics, a wide range of impacting models has been applied to simulate and analyze engineering systems operating within bounded dynamic responses. For example, in wheel-rail impacts of railway coaches [30,32,60], impact noise analyses [56], inertial shakers [20,48], vibrating hammer [57], impact-forming machine [27], offshore structure [51], Jeffcott rotor with bearing clearance [18,38], ground moling dynamics [40], impact dampers [2,11,12,50] and gears [17,21], etc., impacting models have proved to be useful. It is important to note, that most studies of vibro-impact dynamics have been carried out for vibratory systems with elastic impacts in the past several years. However, as it has been mentioned, very few [46,47,52,53,57] have considered plastic impact oscillators. Periodic motion and bifurcations of single-degree-of-freedom oscillators with plastic impacts were investigated by Shaw [46,47] and Xie [57]. Wagg [52,53] studied periodic sticking motion and multi-sliding bifurcation in a two-degree-of-freedom impact oscillator. However, such systems with plastic impacts have a wide range of practical applications as, for example, inertial shakers, pile drivers, milling and forming machines, etc. To fill this gap, a detailed mathematical modeling and non-linear dynamic analysis of a small vibro-impact pile driver is given in this paper. The model includes the sticking properties of the contact between the driver and the pile.

The small vibro-impact pile driver is a two-degree-of-freedom vibratory system with plastic impacts. Dynamics of the small vibro-impact pile driver can be analyzed by means of a three-dimensional map, which describes free flight and sticking solutions of the vibro-impact system, between impacts, supplemented by transition conditions at the instants of impacts. Piecewise property and singularity are found to exist in the Poincaré map. The piecewise property is caused by the transitions of free flight and sticking motions of driver and pile immediately after the impact, and the singularity of map is generated via the grazing contact of driver and pile and corresponding instability of periodic-impact motions. These properties of the map have been shown to exhibit particular types of sliding and grazing bifurcations of periodic-impact motions under parameter variation. The influence of *non-standard* bifurcations and parameter variation on the performance of the vibro-impact system is elucidated, and the global bifurcation diagrams are obtained. The global bifurcation diagrams for the impact velocity of the driver versus the forcing frequency enable the practicing engineer to select excitation frequency ranges in which stable single-impact periodic responses can be expected to occur, and to predict the peak-impact velocities of such responses. With this model we can predict much of the qualitative behavior of the actual physical system.

2. Model of small pile driver

The working schematic for a small vibro-impact pile driver is shown in Fig. 1 [58], M_1 is the mass of the pile (including the shelf of pile driver), K_1 is the soil stiffness, supporting the vibratory system, and C_1 is the