Available online at www.sciencedirect.com
ScienceDirect
Nonlinear Analysis: Real World Applications 9 (2008) 1378 – 1393
Nonlinear Analysis
Real World Applications
www.elsevier.com/locate/na

Chaos and chaos synchronization for a non-autonomous rotational machine systems

Yan-Dong Chu[a], Jian-Gang Zhang[a,*], Xian-Feng Li[a,b], Ying-Xiang Chang[a,b], Guan-Wei Luo[a]

[a]School of Mathematics, Physics and Software Engineering, Lanzhou Jiaotong University, Lanzhou 730070, People's Republic of China
[b]Nonlinear Science Research Center, Lanzhou Jiaotong University, Lanzhou 730070, People's Republic of China

Received 6 December 2006; accepted 12 March 2007

Abstract

Chaos and chaos synchronization of the centrifugal flywheel governor system are studied in this paper. By mechanics analyzing, the dynamical equation of the centrifugal flywheel governor system is established. Because of the non-linear terms of the system, the system exhibits both regular and chaotic motions. The characteristic of chaotic attractors of the system is presented by the phase portraits and power spectra. The evolution from Hopf bifurcation to chaos is shown by the bifurcation diagrams and a series of Poincaré sections under different sets of system parameters, and the bifurcation diagrams are verified by the related Lyapunov exponent spectra. This letter addresses control for the chaos synchronization of feedback control laws in two coupled non-autonomous chaotic systems with three different coupling terms, which is demonstrated and verified by Lyapunov exponent spectra and phase portraits. Finally, numerical simulations are presented to show the effectiveness of the proposed chaos synchronization scheme.

Keywords: Centrifugal governor; Lyapunov exponents; Hopf bifurcation; Periodic motion; Chaos

1. Introduction

During the last two decades, synchronization in chaotic dynamic systems has received a great deal of interest among scientists from various research fields [3,7,4,8,12,20,25,27,31] since Pecora and Carroll [28] introduced a method to synchronize two identical chaotic systems with different initial conditions. Pecora and Carroll showed that there exists the class of chaotic systems for which synchronization can be achieved. Consider a system that can be divided into the drive subsystem (whose largest Lyapunov exponent is positive) and the driven subsystem (with all negative Lyapunov exponents). In this case trajectories from two identical driven subsystems can be synchronized if the same driven system is used. This result has been numerically and experimentally verified mainly on electrical systems. de Sousa Vieira et al. [30] showed that the boundary of possible synchronization and non-synchronization is strictly connected with the transition from chaotic to hyperchaotic behavior that is characterized by at least two positive Lyapunov exponents.

* Corresponding author. Tel./fax: +86 931 4938401.
E-mail addresses: cyd@mail.lzjtu.cn (Y.-D. Chu), zhangjg7715776@126.com (J.-G. Zhang), lixf1979@126.com (X.-F. Li), changyx@mail.lzjtu.cn (Y.-X. Chang).

1468-1218/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.nonrwa.2007.03.009

Nowadays, different techniques and methods have been proposed to achieve chaos control and chaos synchronization such as linear and non-linear feedback approach [1,15,35,36], PC method [28], OGY method [24], adaptive method [32], time-delay feedback approach [27], etc. On the other hand, when the chaotic systems have some uncertain parameters, it is generally difficult to control the system. Therefore, the derivation of an adaptive controller for the control and synchronization of chaotic systems in the presence of unknown system parameters is an important issue [9,14,26]. In this regard, the adaptive feedback synchronization for several chaotic systems has been investigated by Wang et al. [32], Han et al. [14], Elabbasy et al. [9], Lu et al. [19], and Park [25]. Nevertheless, the research on chaos synchronization has intensively focused on the autonomous chaotic systems. Recently, many non-autonomous chaotic systems have been discovered in engineering and life science [5,6,11,13,17,34], and their synchronization has been discussed in Refs. [11,6,34]. In this paper, we numerically examined that two identical rotational machines coupled, respectively, by linear, sinusoidal and exponential state error feedback controllers can achieve chaos synchronization.

The centrifugal governor [2] is a device that automatically controls the speed of an engine and prevents the damage caused by sudden change of load torque. It plays an important role in many rotational machines such as diesel engine, steam engine and so on. In this paper, the complex dynamic behaviors of the centrifugal flywheel governor system are studied further. A number of numerical analyses, such as bifurcation diagram, phase portraits, Poincaré map and Lyapunov exponent, are used to study the dynamical behaviors of the mechanical centrifugal flywheel governor system. For a broad range of parameters, the Lyapunov exponent is the most powerful method to measure the sensitivity of the dynamical system to change in initial conditions. It can help us to examine whether the system is in chaotic motion or not.

The organization of this paper is as follows: In Section 2, we present a method for calculating Lyapunov exponents and Lyapunov dimensions from time series date. In Section 3, we set the notations and derive the equations needed, together with the parameter values chosen for simulations in Section 4. In Section 5, the mechanical centrifugal flywheel governor system is treated as a numerical example to demonstrate the effectiveness of the proposed chaos synchronization method. Numerical simulations are provided to illustrate the performance of the proposed control strategy. Finally concluding remark is given in Section 6.

2. Compute Lyapunov exponents and Lyapunov dimensions from time series date

Lyapunov exponents quantify the average exponential separation between nearby phase space trajectories. An exponential divergence of initially nearby trajectories in phase space coupled with folding of trajectories (to ensure that solutions remain finite) is the generic mechanism for generating deterministic randomness and unpredictability. Indeed, the existence of a positive Lyapunov exponent for almost all initial conditions in a bounded dynamical system is a widely used definition of deterministic chaos.

Let $\vec{x}_0(t)$ denote a reference trajectory passing through $\vec{x}_0(0)$ at time $t=0$ and let $\vec{x}_1(t)$ denote a trajectory passing through $\vec{x}_1(0)$ at time $t=0$. The (maximum) Lyapunov exponent $\lambda(\vec{x}_0)$ defined with respect to the reference orbit $\vec{x}_0$ by [23,33] is

$$\lambda(\vec{x}_0)=\lim_{t\to\infty}\lim_{\|\Delta\vec{x}(0)\|\to 0}\frac{1}{t}\log\frac{\|\Delta\vec{x}(t)\|}{\|\Delta\vec{x}(0)\|}, \tag{1}$$

where $\|\Delta\vec{x}(0)\|$ is the Euclidean distance between the trajectories $\vec{x}_0(t)$ and $\vec{x}_1(t)$ at an initial time $t=0$, and $\|\Delta\vec{x}(t)\|$ is the Euclidean distance between the trajectories $\vec{x}_0(t)$ and $\vec{x}_1(t)$ at a later time t. In this definition $\vec{x}_1(t)$ can be any trajectory that is initially infinitesimally close to $\vec{x}_0(0)$ at time $t=0$. The correspondence between sensitivity to initial conditions and a positive Lyapunov exponent is obvious in the rearrangement

$$\|\Delta\vec{x}(t)\|\sim\|\Delta\vec{x}(0)\|e^{\lambda t}. \tag{2}$$

A dynamical system in $\mathbf{R}^m$ has associated with it m Lyapunov exponents

$$\lambda_1\geqslant\lambda_2\geqslant\cdots\geqslant\lambda_m. \tag{3}$$

To define the full set of exponents, consider an infinitesimal m-dimensional sphere of initial conditions that is anchored to a reference trajectory. As the sphere evolves, it becomes deformed into an ellipsoid. Let $p_i(t)$ denote the length of