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Global exponential stability in Lagrange sense for recurrent neural networks with time delays

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Abstract

In this paper, we study the global exponential stability in Lagrange sense for continuous recurrent neural networks (RNNs) with multiple time delays. Three different types of activation functions are considered, which include both bounded and unbounded activation functions. By constructing appropriate Lyapunov-like functions, we provide easily verifiable criteria for the boundedness and global exponential attractivity of RNNs. These results can be applied to analyze monostable as well as multistable neural networks.

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1. Introduction

Recurrent neural networks (RNNs) have found many applications since the pioneering work of Hopfield [10]. In employing RNNs to solve optimization, control, or signal processing problems, one of the most desirable properties of RNNs is the Lyapunov global stability. From a dynamical system point of view, globally stable networks in Lyapunov sense are monostable systems, which have a unique equilibrium attracting all trajectories asymptotically. A large body of research now exist on the study of global asymptotic stability for RNNs. We refer to [4,5,13–15,17–19,21,23,33,35,36] and the references therein for detailed mathematical analysis on global convergence of various neural network models.

In many other applications, however, monostable neural networks have been found computationally restrictive and multistable dynamics are essential to deal with important neural computations desired. For example, in a winner-take-all network [27,31], where, depending on the external input (or the initial value), only the neuron with the strongest input (or highest initial value) should remain active. This is possible only if there are multiple equilibria with some being unstable. When a neural network is used as an associative memory storage or for pattern recognition, the existence of

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many equilibria is also necessary [3,6,10,24]. In these applications, the neural networks are no longer globally stable, and more appropriate notions of stability are needed to deal with multistable systems.

Motivated by Yi and Tan [33], who take the view that boundedness, attractivity and complete convergence are three basic properties of a multistable network, we study the first two properties in this paper for RNNs. More specifically, we generalize the conventional notion of stability in Lyapunov sense and study the global exponential stability (GES) in Lagrange sense for RNNs with time delay. Our work extends the previous results obtained by Liao and Wang [18], who investigate the global dissipativity and global exponential dissipativity of neural networks. By constructing appropriate Lyapunov-like functions, we provide easily verifiable criteria for the boundedness of the networks and the existence of globally exponentially attractive (GEA) sets. Three different types of neuron activation functions are considered, which include both bounded and unbounded activation functions. Because we do not make any assumptions on the number of equilibria, our results can be used in analyzing both monostable and multistable networks. Once a network is proved to be globally expontentially stable in Lagrange sense, one need only to focus the study of its dynamics inside the (compact) attractive set, where stable and unstable equilibria, periodic orbits, or even chaotic attractors may coexist. These more complex and richer dynamics are essential for the networks to possess multiple stable patterns that are often important in practical applications [1,2,7,8,22,37,38,40]. In the special case where the compact attractive set is a single point, the network is then globally stable in Lyapunov sense and the attractive set is the unique equilibrium point.

It is worth to mention that unlike Lyapunov stability, Lagrange stability refers to the stability of the total system, not the stability of equilibrium points. The boundedness of solutions and the existence of global attractive sets lead to a total system concept of stability: (asymptotic) Lagrange stability. Our paper extends this concept of stability to neural networks and consider it as a compatible method for assessing the stability of multistable neural networks and their mathematical models.

We note that Lagrange stability has long been studied in the theory and applications of dynamical systems. In [11,34], LaSalle and Yoshizawa apply Lyapunov functions to study Lagrange stability. In [26], Rekasius consider asymptotic stability in Lagrange sense for nonlinear feedback control systems. In [28], Lagrange stability is discussed by Thornton and Mulholland as a useful concept for determining the stability of ecological systems. More recently, Passino and Burgess [25] adapt the concept of Lagrange stability to investigate discrete event systems, and Hassibi et al. [9] study the Lagrange stability of hybrid dynamical systems. See also [30,32] for recent results on Lagrange stability for pendulum-like systems.

This paper is organized as follows. In Section 2, we define the notion of GES in Lagrange sense and give two preliminary results that will be used in the proofs of the main results. Section 3 provides several sufficient conditions for the GES of neural networks with bounded (not necessarily monotone) activation functions. GES in the case of Lurie-type activation functions are studied in Section 4. Section 5 considers GES of RNNs with general monotone activation functions. Finally, in Section 6, we give several examples to illustrate the applications of the results.

2. Preliminaries

Consider the following recurrent neural network (CNN) with multiple time delays

$$c_i \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -\mathrm{d}_i x_i(t) + \sum_{j=1}^{n} (\tilde{a}_{ij} g_j(x_j(t)) + b_{ij} g_j(x_j(t-\tau_j))) + I_i(t), \tag{1}$$

where $i = 1, 2, \ldots, n$, $c_i > 0$ and $d_i > 0$ are constants, $x_i(t)$ is the state variable of the ith neuron, $I_i(t) \in C(\mathbb{R}, \mathbb{R})$ is a variable input (bias), $\tilde{a}_{ij}$ and b_{ij} are connection weights from neuron i to neuron j, $\tau_j \geqslant$ are constant time delays, and $g_i \in C(\mathbb{R}, \mathbb{R})$ is the ith neuron activation function. When all $b_{ij} = 0$, or when there are no time delays, the neural network model (1) reduces to the following neural network of ordinary differential equations:

$$c_i \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -d_i x_i(t) + \sum_{j=1}^{n} \tilde{a}_{ij} g_j(x_j(t)) + I_i(t), \quad i = 1, 2, \ldots, n. \tag{2}$$