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New results on input-to-state convergence for recurrent neural networks with variable inputs

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Abstract

In this paper, we study a class of recurrent neural networks (RNNs) arising from optimization problems. By constructing appropriate Lyapunov functions, we prove two new results on input-to-state convergence of RNNs with variable inputs. Numerical simulations are also given to demonstrate the convergence of the solutions.

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1. Introduction

Consider the following recurrent neural network (RNN) model of nonlinear differential equations

$$\frac{dx_i(t)}{dt} = \sum_{j=1}^{n} a_{ij} g_j(x_j(t)) + I_i(t), \quad i = 1, 2, \ldots, n, \quad t \geqslant 0, \tag{1}$$

where $a_{ij}, i, j = 1, 2, \ldots, n$, are constant connection weights and $A = (a_{ij})_{n\times n}$ is the connection matrix, the inputs $I_i(t), i = 1, 2, \ldots, n$, are continuous functions defined on $[0, +\infty)$, and $g_i(s)$, $i = 1, 2, \ldots, n$, are the activation functions of the network. In many applications, the activation functions $g_i(s)$ often take the form of sigmoid functions.

The neural network model of the form (1) is different from the well-known Hopfield model as well as the bidirectional associative memory (BAM) and cellular neural network models ([8]). It does not have a linear term, and the inputs $I_i(t)$ are functions of t. This class of RNN models have found many applications in solving optimization problems (see [4–7]) and has been extensively studied in the monograph [8].

For the neural network model (1), an important concept, called input-to-state convergence (ISC), has been proposed and studied in [8]. The ISC concept is similar to the widely recognized notion of input-to-state stability (ISS), which has been a useful concept in studying nonlinear control problems (see [2,3]). In this paper, we continue the study of ISC for the recurrent neural network model (1) and prove two new results. The first result gives new criteria that are

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not seen in the literature. The second result, which is proved using a different and simpler approach, generalizes and improves Theorem 7.2 from [8]. Examples are also given to illustrate our results.

2. Preliminaries

Let $A=(a_{ij})_{n\times n}$, $I(t)=(I_1(t),I_2(t),\ldots,I_n(t))^{\mathrm{T}}$, $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^{\mathrm{T}}$, and $G(x(t))=(g_1(x_1(t)), g_2(x_2(t)),\ldots,g_n(x_n(t)))^{\mathrm{T}}$. We rewrite (1) in a compact vector form

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t}=AG(x(t))+I(t),\quad t\geqslant 0. \tag{2}$$

Throughout the paper, for a given continuous vector function $I(t)$ and a constant vector I, $(I(t),I)$ will be called an *input pair*. Each activation function g_i will be assumed to be a sigmoid function satisfying

(1) $\lim_{s\to\pm\infty} g_i(s)=\pm 1$, $|g_i(s)|\leqslant 1$, $s\in\mathbb{R}$;
(2) $0<\dot{g}_i(s)<\dot{g}_i(0)$, $s\in\mathbb{R}\backslash\{0\}$;
(3) $\lim_{s\to\pm\infty}\dot{g}_i(s)=0$.

Obviously, for each such function g_i, its inverse g_i^{-1} exists and is continuous. Typical examples of sigmoid functions are

$$\tanh(s)=\frac{\mathrm{e}^s-\mathrm{e}^{-s}}{\mathrm{e}^s+\mathrm{e}^{-s}},\quad \frac{1-\mathrm{e}^{-s}}{1+\mathrm{e}^{-s}},\quad \frac{2}{\pi}\tan^{-1}\left(\frac{\pi}{2}s\right),\quad \frac{1}{1+\mathrm{e}^{-s}}.$$

For any $x(0)\in R^n$, $x(t,x(0))$ will denote the solution of (1) starting from $x(0)$.

The notion of ISC was first introduced and discussed in [8]. Our definition below is adapted from [8] and appears to be more appropriate in relating the ISC of the network to its variable input.

Definition 2.1. The network (1), or equivalently (2), is said to be input-to-state convergent (ISC) with respect to an input pair $(I(t),I)$, if

$$\Omega(I)\triangleq\{x^*\in R^n\,|\,AG(x^*)+I=0\}\neq\emptyset \tag{3}$$

implies that for any $x(0)\in\mathbb{R}^n$, $\lim_{t\to\infty}x(t,x(0))=x^*$ for some $x^*\in\Omega(I)$.

For each $i=1,2,\ldots,n$, define

$$\begin{cases}\underline{g}_i=\inf\limits_{-\infty<s<+\infty} g_i(s),\\ \overline{g}_i=\sup\limits_{-\infty<s<+\infty} g_i(s).\end{cases} \tag{4}$$

It is easily seen (see [8]) that if the inverse $A^{-1}=(\hat{a}_{ij})_{n\times n}$ exists, then the set $\Omega(I)\neq\emptyset$ if and only if

$$-\sum_{j=1}^{n}\hat{a}_{ij}I_j\in(\underline{g}_i,\overline{g}_i),\quad i=1,2,\ldots,n. \tag{5}$$

It is also obvious from the assumed monotonicity of the activation functions that if A^{-1} exists and $\Omega(I)\neq\emptyset$, then $\Omega(I)$ must be a singleton (i.e., a one-point set).

Recall that a matrix $M=(m_{ij})_{n\times n}$ is called an M-matrix if $m_{ii}>0$ $(i=1,2,\ldots,n)$, $m_{ij}\leqslant 0$ $(i\neq j,\ i,j=1,2,\ldots,n)$ and the real part of every eigenvalue of M is non-negative. M is called a non-singular M-matrix if M is both an M-matrix and non-singular.

It can be shown (see [1]) that a matrix M is a non-singular M-matrix if and only if there exist constants $c_i>0$ $(i=1,2,\ldots,n)$ such that

$$c_i m_{ii}+\sum_{j=1,j\neq i}^{n}c_j m_{ij}>0,\quad i=1,2,\ldots,n, \tag{6}$$