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A delayed epidemic model with stage-structure and pulses for pest management strategy

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Abstract

From a biological pest management standpoint, epidemic diseases models have become important tools in control of pest populations. This paper deals with an impulsive delay epidemic disease model with stage-structure and a general form of the incidence rate concerning pest control strategy, in which the pest population is subdivided into three subgroups: pest eggs, susceptible pests, infectious pests that do not attack crops. Using the discrete dynamical system determined by the stroboscopic map, we obtain the exact periodic susceptible pest-eradication solution of the system and observe that the susceptible pest-eradication periodic solution is globally attractive, provided that the amount of infective pests released periodically is larger than some critical value. When the amount of infective pests released is less than another critical value, the system is shown to be permanent, which implies that the trivial susceptible pest-eradication solution loses its attractivity. Our results indicate that besides the release amount of infective pests, the incidence rate, time delay and impulsive period can have great effects on the dynamics of our system.

Keywords: Impulsive control; Delay; Stage structure; Pest management; Attractivity; Permanence

1. Introduction

Pest control strategies were mentioned occasionally in writings of the ancient Chinese, Sumerian, and Egyptian scholars. Predatory ants, for example, were used in China as early as 1200 BC to protect citrus groves from caterpillars and wood boring beetles. A passage in Homer's Iliad (8th century BC) describes the use of fire to drive locusts into the sea, and the ancient Egyptians organized long lines of human drovers to repel swarms of invading locusts. Nowadays, we can choose from many different methods as we plan our strategy for controlling pests.

Compared to chemical treatments, non-chemical methods are safe to man and are generally effective for longer periods of time. One example of non-chemical pest control methods is biological treatment [7,10,11,13,15,19,33,40] including microbial control with pathogens. Entomopathogens are microorganisms that cause disease in arthropods, particularly insects and mites, and naturally widespread in the environment and include bacteria, fungi, viruses, nematodes and protozoa. Most are host specific, and some cause natural epidemics in insect populations. A few species of bacteria

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are highly effective at killing insects. The most important of these is *Bacillus thuringiensis* (Bt). It occurs naturally in insect-rich locations, including soil, plant surfaces and grain stores.

In the nature world, many pests hatch from eggs. For example, Saltcedar leaf beetle is such a pest. In view of its eggshell, pathogens may not be effective against pest eggs. That is, the disease only attacks the susceptible pest population. Also, the time to hatch of the pest egg population (which leads to time lag in our stage structure model) in reasonable pest control policies should not be ignored. To the best of our knowledge, there is a vast amount of literature on the applications of entomopathogens to suppress pests. But there are only a few papers and books on mathematical models of the dynamics of microbial diseases in pest control [2,18,44]. In our work, we introduce the pest based on the stage structure model which incorporates a discrete delay and pulses in order to investigate how epidemics influence the pest control process. We think our model will be a useful experimental tool for determining sensitivities to changes in parameter values, and estimating crucial parameters from data which help us to design practical strategy for pest management.

In recent years, stage structure models have received much attraction [1,4,8,9,28,34,37,41,42,47]. This is not only because they are much more simple than the models governed by partial differential equations but also they can exhibit phenomena similar to those of partial differential models [4], and many important physiological parameters can be incorporated. The single species model with stage structure was studied by Aiello and Freedman [1]. Two species models with stage structure were investigated by Wang and Chen [41], Xiao and Chen [42] and Magnusson [34], Zhang et al. [47], Cui and Song [9], Cui et al. [8] and others. On the other hand, impulsive differential equations [3,14,24] have been recently used in population dynamics in relation to impulsive vaccination [12,17,21,36], population ecology [22,26,29–31,43,45,46], the chemotherapeutic treatment of disease [23], birth pulses [38], the theory of the chemostat [16]. However, impulsive differential equations with time delay have seldom been discussed by authors [25,27,32,35,39] because these types of differential equations, which greatly enrich biological background, are very complicated. In present paper, we propose a new delayed epidemic model with stage structure and impulsive effects which is applied to the study of impulsive delay differential equations. From a pest control point of view, under the assumption that infectious pests do not attack crops, our aim is to keep all susceptible pests at an acceptably low level (below the economic threshold level (ETL)) by releasing infectious pests: not to make all pests be infective ones, only to control susceptible pests with an appropriate use of the control variable (the release amount of infective pests). We answer the following crucial questions: How many infective pests do we release to control the susceptible pest population? How do we evaluate the maximum period of an impulsive effect according to the parameters of the system?

The organization of this paper is as follows. In the next section, we derive the model and introduce some essential hypotheses. To prove our main results we also give several definitions, notations and lemmas. In Section 3, we establish a sufficient condition for the global attractivity of the susceptible pest-eradication periodic solution. The sufficient condition for the permanence of the model is obtained in Section 4. In the final section, we interpret our results in terms of their ecological implications. Also we present some numerical experiments to illustrate the results. Finally we point out some future research directions.

2. Model formulation and preliminary results

In this paper, the pest population is divided into egg, susceptible and infective classes, with the size of each class given by $x_1(t)$, $x_2(t)$ and $y(t)$, respectively, and we introduce the problem we shall study:

$$\begin{cases} \left.\begin{array}{l} \dot{x}_1(t) = B(x_2(t))x_2(t) - \gamma x_1(t) - \mathrm{e}^{-\gamma\tau} B(x_2(t-\tau))x_2(t-\tau), \\ \dot{x}_2(t) = \mathrm{e}^{-\gamma\tau} B(x_2(t-\tau))x_2(t-\tau) - \beta x_2(t) f(y(t)) - \eta x_2(t), \\ \dot{y}(t) = \beta x_2(t) f(y(t)) - wy(t), \end{array}\right\} \quad t > 0, \quad t \neq kT, \\ \Delta y(t) = \mu, \quad t = kT, \quad k \in \mathbf{N} \doteq \{1, 2, \ldots\} \end{cases} \tag{1}$$

with initial conditions

$$\begin{cases} (x_1(t), x_2(t), y(t)) = (\varphi_1(t), \varphi_2(t), \varphi_3(t)) \in \mathbf{C}_3^+ \quad \text{for } t \in [-\tau, 0], \varphi_i(0) > 0, \ \ i = 1, 2, 3, \\ \varphi_1(0) = \int_{-\tau}^{0} \mathrm{e}^{\gamma\theta} B(\varphi_2(\theta))\varphi_2(\theta)\,\mathrm{d}\theta, \end{cases} \tag{2}$$

where

$$\mathbf{C}_3^+ \doteq C\left([-\tau, 0], \mathbf{R}_+^3\right),$$