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Chaos synchronization using fuzzy logic controller☆

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Abstract

The design of a rule-based controller for a class of master-slave chaos synchronization is presented in this paper. In traditional fuzzy logic control (FLC) design, it takes a long time to obtain the membership functions and rule base by trial-and-error tuning. To cope with this problem, we directly construct the fuzzy rules subject to a common Lyapunov function such that the master–slave chaos systems satisfy stability in the Lyapunov sense. Unlike conventional approaches, the resulting control law has less maximum magnitude of the instantaneous control command and it can reduce the actuator saturation phenomenon in real physic system. Two examples of Duffing–Holmes system and Lorenz system are presented to illustrate the effectiveness of the proposed controller.

Keywords: Chaos synchronization; Fuzzy logic control; Lyapunov function; Duffing–Holmes system; Lorenz system

1. Introduction

Over the past decades, chaos plays a more and more important role in nonlinear science field [1,5,18,12]. Synchronization of chaotic systems has received a significant attention, since Pecora and Carroll presented the chaos synchronization method to synchronize two identical chaotic systems with different initial values in 1990 [14]. Generally the two chaotic systems in synchronization are called drive system and response system, respectively. Nowadays, chaos and its synchronization have found a lot of useful applications in many fields of engineering and science such as in secure communications, chemical reactions, power converters, biological systems, and information processing, etc. [3]. Many methods have been presented for the control and synchronization of chaotic system such as periodic parametric perturbation method [2], drive-response synchronization method [24], adaptive control method [21,4,10,9,22], variable structure (or sliding mode) control method [6,28,29], backstepping control method [20,11], and H^{∞} control method [17], among many others [3].

Generally speaking, the synchronization phenomenon has the following feature: the output of the drive system is used to control the response system so that the output of the response system follows the output of the drive system asymptotically. In recent years, some chaos synchronizations based on fuzzy systems have been proposed [19]. The fuzzy set theory was initiated by Zadeh [30]. The fuzzy logic control (FLC) schemes have been widely developed for almost 40 years, and have been successfully applied to many applications [8]. Besides, the adaptive fuzzy controller had also been used to control and synchronize the chaotic systems [7,23]. However, it takes a long time to obtain the

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membership functions and rule base by trial-and-error tuning in traditional FLC design. The concept of robust FLC design process is still never used to control and synchronize the chaotic systems in my mind.

The objective of this paper is to propose a robust fuzzy logic controller to force the master–slave n-dimensional chaotic systems to be synchronized even if they have differential initial conditions, system uncertainties and external disturbances. A set of the fuzzy linguistic rules based on expert knowledge are used to design the control law of FLC. To overcome the trail-and-error tuning of the membership functions and rule base, we directly construct the fuzzy rules subject to a common Lyapunov function such that the error dynamics satisfies stability in the Lyapunov sense. Simulation results show that the proposed controller drives the slave system to synchronize to the master system in spite of different initial conditions, system uncertainties and external disturbances. Besides, the chaos synchronization systems are synchronized, without respect to the error dynamics are standard forms (Duffing–Holmes systems) or nonstandard forms (Lorenz systems). The organization of this paper is as follows: Section 2 addresses the original and design models of the master–slave chaos synchronization system. Section 3 describes the design approaches of FLC for Duffing–Holmes systems and Lorenz systems. Finally, conclusions are given in Section 4.

2. System description and problem formulation

In this paper, we consider a class of the following two n-dimensional chaotic systems,

$$\begin{cases} \dot{x}_i = x_{i+1}, & 1 \leqslant i \leqslant n-1, \\ \dot{x}_n = f(x,t), & \end{cases} \quad x = [x_1, x_2, \ldots, x_n] \in R^n, \tag{1}$$

$$\begin{cases} \dot{y}_i = y_{i+1}, & 1 \leqslant i \leqslant n-1, \\ \dot{y}_n = f(y,t) + \Delta f(y) + d(t) + u, & \end{cases} \quad y = [y_1, y_2, \ldots, y_n] \in R^n, \tag{2}$$

where $u \in R$ is the control input, f is a given nonlinear function of x and t, $\Delta f(y)$ is an uncertain term representing the unmodeled dynamics or structural variation of system (2) and $d(t)$ is the disturbance of system (2). System (1) is most often applied in physical systems such as the Duffing–Holmes damped spring system, Van der Pol equation, Genesio system [13], robot systems and flexible-joint mechanisms [16]. It is also assumed that $f(x,t)$ and $f(y,t)$ satisfy all the necessary conditions, such as systems (1) and (2) having unique solution in the time interval $[t_0, +\infty)$, $t_0 > 0$, for any given initial condition $x_0 = x(t_0)$ and $y_0 = y(t_0)$. The dynamics of system (1) display chaotic motion without control input ($u = 0$).

The control problem considered in this paper is that for different initial conditions of systems (1) and (2), the two coupled system, i. e. the master system (1) and the slave system (2), to be synchronized by designing an appropriate control $u(t)$ in system (2) such that

$$\lim_{t\to\infty} \|x(t) - y(t)\| \to 0, \tag{3}$$

where $\|\cdot\|$ is the Euclidean norm of a vector.

Let us define the state errors between the master system and slave system as

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \ldots, e_n = y_n - x_n. \tag{4}$$

The error dynamic system can be derived into a standard form as

$$\begin{cases} \dot{e}_1 = e_2, \\ \dot{e}_2 = e_3, \\ \vdots \\ \dot{e}_{n-1} = \dot{e}_n, \\ \dot{e}_n = g(x_1, x_2, \ldots, x_n, e_1, e_2, \ldots, e_n) + \Delta f(e+x) + d(t) + u, \end{cases} \tag{5}$$

where g is a nonlinear function. The problem to realize the synchronization between two chaotic system now is transformed to another problem how to choose a control law $u(t)$ to make error states e_i $(i = 1, 2, \ldots, n)$ in (5) generally converge to zero. Here robust fuzzy control design is used to achieve the objective.