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Asymptotic behavior of an invading species

Antonio Tineo*

Departamento de Matemáticas, Facultad de Ciencias, Universidad de los Andes, Merida, Venezuela

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Abstract

In this paper we consider $n+1$ competing species such that n of them are permanent and we study the behavior of the rest species. Our study complements a work initiated by S. Ahmad and A.C. Lazer concerning the extinction or persistence of the rest species.

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1. Introduction

In [2], Ahmad and Lazer consider the Lotka–Volterra system

$$x_i' = x_i\left[a_i(t) - \sum_{j=1}^{n} b_{ij}x_j - c_i y\right], \quad 1 \leqslant i \leqslant n,$$

$$y' = y\left[\alpha(t) - \sum_{j=1}^{n} q_j x_j - \beta y\right], \tag{1.1}$$

where $a_1, \ldots, a_n, \alpha : \mathbb{R} \to \mathbb{R}$ are bounded continuous functions and b_{ij}, q_j, β are nonnegative constants such that $\beta, b_{ii} > 0$ for all i. They also assume,

(AL$_1$) $\inf(a_1), \ldots, \inf(a_n), \inf(\alpha) > 0$.
(AL$_2$) $a_1, \ldots, a_n, \alpha$ possesses average $[a_1], \ldots, [a_n], [\alpha]$ in the sense of the definition below. See also [2].
(AL$_3$)

$$\inf(a_i) > \sum_{j=1, j\neq i}^{n} \frac{b_{ij}}{b_{jj}} \sup(a_j) + \frac{c_i}{\beta} \sup(\alpha), \quad 1 \leqslant i \leqslant n.$$

* Tel.: +58 274 4160286; fax: +58 274 401286.
E-mail address: atineo@ula.ve.

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In particular, the system

$$x_i' = x_i\left[a_i(t) - \sum_{j=1}^{n} b_{ij}x_j\right], \quad 1 \leqslant i \leqslant n, \tag{1.2}$$

has a global attractor $u^* = (u_1^*, \ldots, u_n^*)$ such that $0 < \inf(u_i^*) \leqslant \sup(u_i^*) < \infty$, for all i. See [1]. Moreover, each u_i^* possesses average. See [2].

Under these conditions, they prove:

(TAL_1) If

$$[\alpha] < \sum_{j=1}^{n} q_j[u_j^*]$$

then, the species y goes to be extinct. That is, $\lim_{t\to\infty} y(t)=0$, for any positive solution $(x, y) := (x_1, \ldots, x_n, y)$ of the system.

(TAL_2) If

$$[\alpha] > \sum_{j=1}^{n} q_j[u_j^*],$$

then system (1.1) is persistent. That is, $\liminf_{t\to\infty} y(t) > 0$, for any positive solution (x, y) of the system.

Ahmad and Lazer, in the above mentioned paper, have conjectured that the above results remain true if we replace condition (AL_3) by

$$[a_i] > \sum_{j=1, j\neq i}^{n} \frac{b_{ij}}{b_{jj}}[a_j] + \frac{c_i}{\beta}[\alpha], \quad 1 \leqslant i \leqslant n.$$

The main result that the present paper implies that this conjecture holds true. In fact, using the concept of lower and upper average defined by the author in [5] and a decomposition lemma given by Ortega and Tineo [3], we will prove

Theorem 1.1. *Assume $[a_i]_L > 0$ for all i and $[\alpha]_L > 0$. Assume further that*

$$[a_i]_L > \sum_{j=1, j\neq i}^{n} \frac{b_{ij}}{b_{jj}}[a_j]_U + \frac{c_i}{\beta}[\alpha]_U, \quad 1 \leqslant i \leqslant n, \tag{1.3}$$

and let (x, y) be a positive solution of (1.1).

(a) *If*

$$\left[\alpha - \sum_{j=1}^{n} q_j u_j^*\right]_U < 0,$$

then

$$\lim_{t\to+\infty} y(t) = 0.$$

(b) *If*

$$\left[\alpha - \sum_{j=1}^{n} q_j u_j^*\right]_L > 0,$$