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Multiple positive periodic solutions for a generalized predator–prey system with exploited terms☆

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Abstract

In this paper, we consider a generalized predator–prey system with exploited terms and prove the existence of eight positive periodic solutions by employing the continuation theorem of coincidence degree theory.

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1. Introduction

Mukherjee et al. [3] considered and investigated a generalized model representing two predators for a prey with a delay due to gestation, which is of the form:

$$\begin{cases} \dfrac{dx_1(t)}{dt} = x_1(t)g(x_1(t)) - c_1x_2(t)p_2(x_1(t)) - c_2x_3(t)p_3(x_1(t)), \\ \dfrac{dx_2(t)}{dt} = x_2(t)[-d_1 + \beta_2 p_2(x_1(t-\tau)) - a_{22}x_2(t)], \\ \dfrac{dx_3(t)}{dt} = x_3(t)[-d_2 + \beta_3 p_3(x_1(t-\tau)) - a_{33}x_3(t)], \end{cases} \tag{1.1}$$

where $\tau > 0$ is a constant time delay, all parameters are positive numbers, $x_1(t)$ denotes the biomass at time t of the resource species and $x_2(t)$ and $x_3(t)$ represent predator densities. There is a delay of time τ due to gestation. $g(x_1)$ is the specific growth rate of the resource species; $p_i(x_1)$, $i = 2, 3$ are the functional responses for the ith predator; a_{22} and a_{33} denote the intraspecific competition coefficients of the predators; β_i, $i = 2, 3$ are the conversions of biomass constant and d_i, $i = 1, 2$ are death rates of the predators. For biological relevance of the system, see Mukherjee et al. [3].

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In Mukherjee et al. [3], they derived condition for the global stability in terms of the parameters of system to guarantee it.

Zhang and Wang [5], recently, are concerned with the nonautonomous form of system (1.1), that is to consider the following generalized predator–prey system:

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = x_1(t)g(t, x_1(t)) - c_1(t)x_2(t)p_2(x_1(t)) - c_2(t)x_3(t)p_3(x_1(t)), \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = x_2(t)[-d_1(t) + \beta_2(t)p_2(x_1(t-\tau)) - a_{22}(t)x_2(t)], \\ \dfrac{\mathrm{d}x_3(t)}{\mathrm{d}t} = x_3(t)[-d_2(t) + \beta_3(t)p_3(x_1(t-\tau)) - a_{33}(t)x_3(t)], \end{cases} \tag{1.2}$$

where $\tau > 0$ is the same as that in system (1.1), parameters are positive continuous periodic functions with period $\omega > 0$. In [5], we first establish the existence result of a positive periodic solution for system (1.2) by using the continuation theorem of coincidence degree theory. Then by constructing a Lyapunov V functional, we establish the global attractivity of a positive periodic solution for system (1.2).

In the present paper, we are concerned with the following generalized predator–prey system with multiple exploited terms [4,2]:

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = x_1(t)g(t, x_1(t)) - c_1(t)x_2(t)p_2(x_1(t)) - c_2(t)x_3(t)p_3(x_1(t)) - h_1(t), \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = x_2(t)[-d_1(t) + \beta_2(t)p_2(x_1(t-\tau)) - a_{22}(t)x_2(t)] - h_2(t), \\ \dfrac{\mathrm{d}x_3(t)}{\mathrm{d}t} = x_3(t)[-d_2(t) + \beta_3(t)p_3(x_1(t-\tau)) - a_{33}(t)x_3(t)] - h_3(t), \end{cases} \tag{1.3}$$

where $\tau > 0$ is constant, parameters themselves are all positive continuous ω-periodic functions; $g(t, x_1) : R \times R \to R$ is continuous and ω-periodic with respect to t, $p_i : R \to R$ are continuous, $i = 2, 3$, $g(t, x_1)$ is monotonously decreasing in x_1 for fixed t, $\forall t, x_1 \in R$, $g(t, 0) > 0$, $\forall t \in R$, $p_i(x)$ are monotonously increasing in x, $i = 2, 3$, $p_i(x) > 0$ for $x > 0$, $i = 2, 3$.

Instead of investigating the existence of a periodic solution, in this paper, we investigate the existence of multiple positive periodic solutions for system (1.3) since the existence results of multiple periodic solutions are very scarce in the literature. The paper is organized as follows. In Section 2, we establish the existence of multiple positive periodic solutions for system (1.3) by using the continuation theorem of coincidence degree theory. In Section 3, we illustrate our result with an example.

2. The existence of eight positive periodic solutions

In this section, based on Mawhin's continuation theorem, we shall study the existence of at least eight positive periodic solutions of system (1.3). First, we shall make some preparations.

Let X, Z be Banach spaces, L: $Dom\, L \subset X \to Z$ be a linear mapping and $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $dim\, Ker\, L = codim\, Im\, L < \infty$ and $Im\, L$ is closed in Z. If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $Im\, P = Ker\, L$ and $Im\, L = Ker\, Q = Im(I - Q)$. It follows that $L/_{Dom\, L \cap Ker\, P} : (I - P)X \to Im\, L$ is invertible. We denote the inverse of the map $L/_{Dom\, L \cap Ker\, P}$ by K_p. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $(QN)(\bar{\Omega})$ is bounded and $K_p(I - Q)N : \bar{\Omega} \to X$ is compact. Since $Im\, Q$ is isomorphic to $Ker\, L$, there exists an isomorphism $J : Im\, Q \to Ker\, L$.

In the proof of our existence theorem, we will use the continuation theorem of Gaines and Mawhin [1].

Lemma 2.1 (*Continuation theorem*). *Let L be a Fredholm mapping of index zero and let N be L-compact on* $\bar{\Omega}$. *Suppose*

(a) $Lx \neq \lambda N(x, \lambda), \forall \lambda \in (0, 1), x \in \partial\Omega$;
(b) $QN(x, 0) \neq 0, \forall x \in Ker\, L \cap \partial\Omega$;
(c) *Brouwer degree* $deg_B(JQN(\cdot, 0), \Omega \cap Ker\, L, 0) \neq 0$.

Then $Lx = Nx$ *has at least one solution in* $Dom\, L \cap \bar{\Omega}$.