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Dynamic behaviors of the periodic predator–prey model with modified Leslie-Gower Holling-type II schemes and impulsive effect☆

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Abstract

In this paper, a predator–prey system which based on a modified version of the Leslie–Gower scheme and Holling-type II scheme with impulsive effect are investigated, where all the parameters of the system are time-dependent periodic functions. By using Floquet theory of linear periodic impulsive equation, some conditions for the linear stability of trivial periodic solution and semi-trivial periodic solutions are obtained. It is proved that the system can be permanent if all the trivial and semi-trivial periodic solutions are linearly unstable. We use standard bifurcation theory to show the existence of nontrivial periodic solutions which arise near the semi-trivial periodic solution. As an application, we also examine some special case of the system to confirm our main results.

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Keywords: Leslie–Gower; Holling-type II; Predator–prey system; Impulsive effect; Bifurcation

1. Introduction

A natural predator–prey system is described as follows:

$$\begin{cases} \dot{x}(t) = ax(t) - \phi(x(t))y(t), \\ \dot{y}(t) = -by(t) + \gamma\phi(x(t))y(t), \end{cases} \tag{1.1}$$

where a, b, γ are positive constants, $\phi(x)$ is the functional response of the predator $y(t)$. Recently the dynamics of a predator–prey system is studied in many papers, for example, Amine et al. [1], Tang [22], López-Gómez et al. [15] and Rinaldi et al. [19]. In this paper, we consider a predator–prey model which incorporates a modified version of the

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Leslie–Gower functional response as well as that of the Holling-type II. About Holling-type II we can see [14], and about Leslie–Gower scheme we can see [2,3,7,12,13,24].

The predator–prey model describes a prey population x which serves as food for a predator y, however, due to seasonal effects of weather, temperature, food supply, mating habits, contact with predators and other resource or physical environmental quantities, we can assume temporal variation to be cyclic or periodic. The rate equations for the two components of system with periodic coefficients can be written as follows:

$$\begin{cases} \dot{x}(t) = \left(r_1(t) - b_1(t)x(t) - \dfrac{a_1(t)y(t)}{x(t)+k_1(t)}\right)x(t), \\ \dot{y}(t) = \left(r_2(t) - \dfrac{a_2(t)y(t)}{x(t)+k_2(t)}\right)y(t), \end{cases} \tag{1.2}$$

where x and y represent the population densities at time t; $b_1(t), r_i(t), a_i(t), k_i(t)$ $(i = 1, 2)$ are model parameters assuming only positive values, $r_1(t)$ is the growth rate of prey x, $b_1(t)$ measures the strength of competition among individuals of species x, $a_1(t)$ is the maximum value which per capita reduction rate of x can attain, $k_1(t)$ (respectively, $k_2(t)$) measures the extent to which environment provides protection to prey x (respectively, to predator y), $r_2(t)$ describes the growth rates of y, and $a_2(t)$ has a similar meaning to $a_1(t)$.

The Leslie–Gower formulation is based on the assumption that reduction in a predator population has a reciprocal relationship with per capita availability of its preferred food. Indeed, Leslie [10] introduced a predator–prey model where the carrying capacity of the predator's environment is proportional to the number of prey. This interesting formulation for the predator dynamics has been discussed by Leslie and Gower in [11] and by Pielou in [17]. It is $\frac{\mathrm{d}y}{\mathrm{d}t} = r_2 y(1 - y/\alpha x)$, in which the growth of the predator population is of logistic form (i.e., $\frac{\mathrm{d}y}{\mathrm{d}t} = r_2 y(1 - y/C)$), but the conventional 'C', which measures the carrying capacity set by the environmental resources is $C = \alpha x$, proportional to prey abundance (α is the conversion factor of prey into predators). The term $y/\alpha x$ of this equation is called Leslie–Gower term. It measures the loss in the predator population due to rarity (per capita y/x) of its favorite food. In the case of severe scarcity, y can switch over to other populations but its growth will be limited by the fact that its most favorite food (x) is not available in abundance. This situation can be taken care of by adding a positive constant d to the denominator. Hence, the equation above becomes $\frac{\mathrm{d}y}{\mathrm{d}t} = r_2 y(1 - y/(\alpha x + d))$, and thus, $\frac{\mathrm{d}y}{\mathrm{d}t} = y(r_2 - (r_2/\alpha)(y/(x + d/\alpha)))$; that is the second equation of system (1.2), $\frac{\mathrm{d}y}{\mathrm{d}t} = (r_2 - a_2 y/(x + k_2))y$; see [2].

The above system can be considered as a representation of an insect pest-spider food chain, nature abounds in systems which exemplify this model; see [24].

However, the ecological system is often affected by environmental changes and other human activities. In many practical situations, it is often the case that predator or parasites are released at some transitory time slots and harvest or stock of the species are seasonal or occur in regular pulses. These short-time perturbations are often assumed to be in the form of impulses in the modelling process. Consequently, impulsive differential equations (hybrid dynamical systems) provide a natural description of such systems, see [9]. Equations of this kind are found in almost every domain of applied sciences. Numerous examples are given in Bainov's and his collaborators' books [4]. They generally describe phenomena which are subject to steep and/or instantaneous changes. Some impulsive equations have been recently introduced in population dynamics, such as vaccination [5,21], chemotherapeutic treatment of disease [8,16], chemostat [6], birth pulse [20,23].

In this paper, we consider the following T-periodic predator–prey system with impulsive effects

$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= \left(r_1(t) - b_1(t)x(t) - \dfrac{a_1(t)y(t)}{x(t)+k_1(t)}\right)x(t), \\ \dot{y}(t) &= \left(r_2(t) - \dfrac{a_2(t)y(t)}{x(t)+k_2(t)}\right)y(t), \end{aligned}\right\} \quad t \neq \tau_k, \quad k \in Z_+, \\ \left.\begin{aligned} x(\tau_k^+) &= (1+h_k)x(\tau_k), \\ y(\tau_k^+) &= (1+g_k)y(\tau_k), \end{aligned}\right\} \quad t = \tau_k, \quad k \in Z_+, \end{cases} \tag{1.3}$$