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Insensitizing controls for a phase field system

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ABSTRACT

In this paper, a nonlinear parabolic system modeling phase field phenomena is considered. This system consists of two coupled parabolic equations, the first one describes the temperature of the material and the second one describes a phase field function. Under small perturbations of the initial data, we study the existence of controls insensitizing the phase field function and acting only on the temperature equation. This problem is equivalent to the null controllability of a parabolic system, which is studied by means of duality arguments, Carleman estimates, and fixed point theorems.

1. Introduction

Motivated by the difficulties in determining data in applications of distributed systems, Lions introduced in [29] the topic of insensitizing controls. This notion deals with the existence of controls making some functional of the solution insensible to small perturbations of the initial data. For some particular functionals, it has been proven that this problem is equivalent to control properties of cascade systems [7,23]. The insensitivity can be defined in an approximate or exact way. Approximate insensitivity is equivalent to approximate controllability of the cascade system, while exact insensitivity is equivalent to its null controllability. The first mathematical results concerned the insensitivity of the L^2-norm of the solution restricted to a subdomain, called the observatory. In [7], approximate results were proven for the heat equation by getting unique continuation properties for parabolic systems. In [14], de Teresa used a global Carleman estimate

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approach to get the existence of exact insensitizing controls for a semilinear heat equation. Other papers on the heat equation are [8], where superlinear nonlinearities were considered, and [23], where the L^2-norm of the gradient of the solution is chosen as the functional to deal with. The same kind of problems have been studied for other systems, as the wave equation [13,2] and fluids equations [22,24,12,11].

In this paper, we address the insensitizing problem for a phase transition model introduced in [9]. We consider a material that may be in two phases, liquid and solid. Instead of studying a free boundary problem, a phase field function is introduced to indicate if the material is in solid or liquid state, varying in a continuous way from one phase to the other. The interface between these two states is supposed to be of finite thickness and defined where the phase field function is not constant. These hypothesis lead to the following nonlinear model (see [9])

$$\begin{cases} y_t - \Delta y = -\ell z_t, \quad \ell > 0, \\ z_t - \Delta z - a(z - z^3) = y, \quad a > 0, \end{cases}$$

describing the evolution of the temperature of the material ($y = y(x,t)$) and the phase field function ($z = z(x,t)$). In [9], well-posedness and asymptotic results (with respect to the thickness of the interface) were proved. From a control point of view, some optimal controls are obtained in [25]. Null controllability of this system is proven in [6] by means of two distributed controls, and in [3,21,4] with only one.

Let us describe in detail the system we are concerned with. Let $T > 0$ and $\omega, \Omega \subset \mathbb{R}^N$, with $N = 2$ or 3, two nonempty bounded open sets such that $\omega \subset \Omega$ with $\partial\Omega$ of class C^2. We define $Q := \Omega \times (0,T)$, $\Sigma := \partial\Omega \times (0,T)$ and $\mathbb{1}_\omega$ the characteristic function of ω. Our phase field system, for a material which occupies the region Ω, is described for the temperature $y = y(x,t)$, and the phase field function $z = z(x,t)$ in the coupled system given by

$$\begin{cases} y_t - d_1\Delta y = -\ell z_t + f_1 + h\mathbb{1}_\omega & \text{in } Q, \\ z_t - d_2\Delta z - (az + bz^2 - z^3) = y + f_2 & \text{in } Q, \\ \dfrac{\partial y}{\partial n} = 0, \qquad \dfrac{\partial z}{\partial n} = 0 & \text{on } \Sigma, \\ y(0) = y_0 + \tau_1\widehat{y}_0, \qquad z(0) = z_0 + \tau_2\widehat{z}_0 & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $d_1, d_2, \ell, a > 0$, $b \in \mathbb{R}$, and the source term h is viewed as a control function supported in ω. Moreover, $f_1, f_2 \in L^2(Q)$ are given external forces. The initial data is composed by a fixed known part given by $y_0, z_0 \in L^2(\Omega)$, and some unknown perturbations given by $\tau_1\widehat{y}_0, \tau_2\widehat{z}_0 \in L^2(\Omega)$. Here, τ_1 and τ_2 are small unknown real numbers and, $\widehat{y}_0, \widehat{z}_0 \in L^2(\Omega)$ are such that $\|\widehat{y}_0\|_{L^2(\Omega)} = \|\widehat{z}_0\|_{L^2(\Omega)} = 1$.

We aim at insensitizing the functional

$$J_{\tau_1,\tau_2}(y,z) = \frac{1}{2}\iint_{\mathcal{O}\times(0,T)} |z|^2 \,\mathrm{d}x\,\mathrm{d}t. \tag{1.2}$$

This means that we look for a control $h \in L^2(0,T;L^2(\omega))$ such that

$$\left.\frac{\partial J_{\tau_1,\tau_2}(y,z)}{\partial \tau_i}\right|_{(\tau_1,\tau_2)=(0,0)} = 0 \quad \forall \widehat{y}_0, \widehat{z}_0 \in L^2(\Omega) \text{ with } \|\widehat{y}_0\|_{L^2(\Omega)} = \|\widehat{z}_0\|_{L^2(\Omega)} = 1,\ i = 1,2. \tag{1.3}$$

The existence of a control satisfying (1.3) is equivalent to the (partial) control problem of finding a function h such that the solution of system

$$\begin{cases} w_t - d_1\Delta w = -\ell u_t + f_1 + h\mathbb{1}_\omega & \text{in } Q, \\ -\theta_t - d_1\Delta\theta = -\ell\theta + q & \text{in } Q, \\ u_t - d_2\Delta u = (a + bu - u^2)u + w + f_2 & \text{in } Q, \\ -q_t - d_2\Delta q = -\ell d_2\Delta\theta - \ell(a + 2bu - 3u^2)\theta + (a + 2bu - 3u^2)q + u\mathbb{1}_{\mathcal{O}} & \text{in } Q, \\ \dfrac{\partial w}{\partial n} = 0, \qquad \dfrac{\partial \theta}{\partial n} = 0, \qquad \dfrac{\partial u}{\partial n} = 0, \qquad \dfrac{\partial q}{\partial n} = 0 & \text{on } \Sigma, \\ w(0) = y_0, \qquad \theta(T) = 0, \qquad u(0) = z_0, \qquad q(T) = 0 & \text{in } \Omega, \end{cases} \tag{1.4}$$