Nonlinear Analysis 143 (2016) 211–223

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

Exponential stability for magneto-micropolar fluids

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A R T I C L E I N F O

Article history:
Received 16 February 2016
Accepted 23 May 2016
Communicated by Enzo Mitidieri

Keywords:
Stability
Magneto-Micropolar fluids
Strong solutions

A B S T R A C T

We prove existence, uniqueness and exponential stability of mildly decaying global strong solutions for the magneto-micropolar fluids system in three space dimensions. Our main objective is to study the convergence of nonstationary solutions to stationary solutions for the system fluids when $t \to \infty$. We show that under mild suitable conditions the convergence is indeed exponential.

1. Introduction

The objective of the present work is to study the exponential stability for strong solutions of the evolution equations governing the motion of incompressible micropolar (asymmetric) fluids in a bounded domain $\Omega \subset \mathbb{R}^3$ having a compact C^2-boundary. To describe these equations, let $T > 0, Q_T \equiv \Omega \times (0,T)$ and $S_T = \partial\Omega \times (0,T)$; the system we are interested in is the following:

$$\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - (\mu+\chi)\Delta\boldsymbol{u} + \nabla\left(\eta + \dfrac{1}{2}r\boldsymbol{h}\cdot\boldsymbol{h}\right) = \chi\,\mathrm{rot}\,\boldsymbol{w} + r\boldsymbol{h}\cdot\nabla\boldsymbol{h} + \boldsymbol{f} \quad \text{in } Q_T, \\ j\dfrac{\partial \boldsymbol{w}}{\partial t} + j(\boldsymbol{u}\cdot\nabla)\,\boldsymbol{w} - \nu_2\Delta\boldsymbol{w} + 2\chi\boldsymbol{w} - \nu_3\,\nabla\mathrm{div}\;\boldsymbol{w} = \chi\mathrm{rot}\;\boldsymbol{u} + \boldsymbol{g} \quad \text{in } Q_T, \\ \dfrac{\partial \boldsymbol{h}}{\partial t} - \nu\Delta\boldsymbol{h} + \boldsymbol{u}\cdot\nabla\boldsymbol{h} - \boldsymbol{h}\cdot\nabla\boldsymbol{u} = 0, \\ \mathrm{div}\,\boldsymbol{u} = 0, \qquad \mathrm{div}\,\boldsymbol{h} = 0, \qquad \text{in } Q_T, \end{cases} \tag{1}$$

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http://dx.doi.org/10.1016/j.na.2016.05.015

together with boundary and initial conditions

$$\begin{cases} \boldsymbol{u} = \boldsymbol{0} \quad \text{on } S_T, \\ \boldsymbol{w} = \boldsymbol{0} \quad \text{on } S_T, \\ \boldsymbol{h} = \boldsymbol{0} \quad \text{on } S_T, \\ \boldsymbol{u}(x,0) = \boldsymbol{u}_0(x) \quad \text{in } \Omega, \\ \boldsymbol{w}(x,0) = \boldsymbol{w}_0(x) \quad \text{in } \Omega, \\ \boldsymbol{h}(x,0) = \boldsymbol{h}_0(x) \quad \text{in } \Omega. \end{cases} \tag{2}$$

The vector-valued functions $\boldsymbol{u} = (u_1, u_2, u_3)$, $\boldsymbol{w} = (w_1, w_2, w_3)$, $\boldsymbol{h} = (h_1, h_2, h_3)$ and the scalar function η denote respectively the linear velocity, the angular velocity of rotation of particles, the magnetic field and the pressure of the fluid. The vector-valued functions $\boldsymbol{f}$ and $\boldsymbol{g}$ denote, respectively, the external sources of linear and angular momentum. The positive constants μ, χ, ν_2, ν_3, j and ν are associated with physical properties of the material. From physical reasons, these constants satisfy $\min\{\mu, \chi, j, \nu, \nu_2 + \nu_3\} > 0$.

Physically, micropolar flows describe the motion of fluids in which the rotation of microparticles is also taken in account. Moreover, when one considers also the effect of an induced magnetic field on the motion, one gets the more complete magnetic-micropolar fluids. The non-Newtonian models of micropolar and magnetic-micropolar fluids have been used in modelling a variety of physical phenomena involving suspensions of rigid particles in fluids, such as human blood, polymeric suspensions, and so on, and therefore have found many applications in physiological and engineering problems. Micropolar fluids exhibit an asymmetric stress tensor, and this is why they are also called asymmetric fluids. See, for more details, Refs. [1,6,10,9,11,15].

We observe that this model of fluid includes as a particular case the classical Navier–Stokes equations, which has been widely studied (see for instance the books by Ladyzhenskaya [13] or Temam [28], and the references therein). This model also includes the micropolar fluid equations as particular case, which also has a large related literature. An excellent general reference on micropolar fluids is the book by Lukaszewicz [15]. There are also several more recent works on the model. For example, in [16], the stationary problem with boundary data in L^2 was studied. In [12], the authors use methods of Clifford analysis to write the system of asymmetric fluids in the hypercomplex formulation and to represent its solution in terms of Clifford operators. The evolution case was studied in [31] through a semigroup approach (see also [30]). Linearization and successive approximations have been considered in [4,19] to give sufficient conditions on the kinematics pressure in order to obtain regularity and uniqueness of the weak solutions to the micropolar fluid equations. More recently, in [14] the authors considered a weak-L^p Prodi–Serrin type regularity criterion. In [17], the existence of restricted global attractors was showed through a semigroup approach. In [5], pointwise time error estimates in suitable Hilbert spaces were shown by considering spectral Galerkin approximations for strong solutions of the micropolar fluid model.

Let us now mention some previous results for the magneto-micropolar system (1)–(2) considered here. For bounded regular domains Ω and initial data similar to the case of classical Navier–Stokes equations, in [26] the authors proved through a spectral Galerkin method the global existence and uniqueness of weak solutions in the two-dimensional case. Existence for local and global in time strong solutions was obtained respectively in [24,18]. In [19], the authors treated uniqueness questions for weak solutions. In [25,20], the authors studied local and uniform in time convergence rates for the approximate solutions constructed by the Galerkin method. The existence of reproductive solutions (so called periodic weak solutions) was studied in [7].

There is no previous work discussing stability of solutions for the system (1)–(2). In the case of the classical Navier–Stokes equations there is abundant literature on this topic though. Prodi [21] was apparently the first to worry about this question in the L^2-context (see also [29]). In [3], the authors studied, in the case $\Omega = \mathbb{R}^3$, the L^p-stability for strong solutions using the ideas developed in [2] (energy estimates). Subsequently, the arguments of [3] were adapted in [22] for the case of bounded domains (see also [32]). More recently, in [23]