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Global weighted estimates in Orlicz spaces for second-order nondivergence parabolic equations

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ABSTRACT

In this paper we prove the following global weighted estimates in Orlicz spaces

$$f \in L_w^\phi(\Omega_T) \Rightarrow u, Du, D^2u, u_t \in L_w^\phi(\Omega_T)$$

for second-order parabolic equations of nondivergence form with small BMO coefficients

$$\begin{cases} u_t - a_{ij}(x,t)u_{x_ix_j} = f & \text{in } \Omega_T, \\ \qquad\qquad u(x,t) = 0 & \text{on } \partial_p\Omega_T. \end{cases}$$

1. Introduction

In this paper we study the following linear parabolic equations of the form

$$u_t - a_{ij}(x,t)u_{x_ix_j} = f \quad \text{in } \Omega_T =: \Omega \times (0,T), \tag{1.1}$$

$$u(x,t) = 0 \quad \text{on } \partial_p\Omega_T, \tag{1.2}$$

where $x = (x_1, x_2, \dots, x_n), i, j = 1, 2, \dots, n$ and the summation convention is understood. Moreover, the coefficients $a_{ij}(x,t)$ satisfy

$$a_{ij} = a_{ji}, \;\; \Lambda^{-1}|\xi|^2 \le a_{ij}(x,t)\xi_i\xi_j \le \Lambda|\xi|^2 \tag{1.3}$$

for all $i, j = 1, 2, \dots, n, \xi \in \mathbb{R}^n$, almost every $(x,t) \in \mathbb{R}^{n+1}$, and a positive constant Λ.

Throughout this paper we assume that the coefficients of $A = \{a_{ij}\}$ are in parabolic BMO space and their parabolic semi-norms are small enough. More precisely, we introduce the following definition.

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Definition 1.1 (*Small BMO Condition*)**.** We say that the matrix A is (δ, R)-vanishing if

$$\sup_{0<r\leq R}\ \sup_{z\in\mathbb{R}^n\times\mathbb{R}} \fint |A(\zeta)-\overline{A}_{Q_r(z)}|d\zeta \leq \delta,$$

where $z=(x,t), \zeta=(y,s)\in\mathbb{R}^n\times\mathbb{R}$, the parabolic cylinder $Q_r=B_r\times(-r^2,r^2]$, and

$$\overline{A}_{Q_r(z)}=\fint A(\zeta)\ d\zeta.$$

L^p-regularity theory, found by Calderón–Zygmund [17], plays an important role in the development of elliptic and parabolic equations. Many authors [1,2,5–7,14–16,18,22,23,28,29,33] extensively studied such estimates for elliptic and parabolic equations with different assumptions on the coefficients and domain. Moreover, weighted Sobolev spaces [3,25,30,36,37] have been extensively studied as the generalization of Sobolev spaces which are sets of functions with a certain degree of smoothness, are commonly used and studied in a wide variety of fields of mathematics, and have turned out to be one of the most powerful tools in analysis created in the 20th century. Furthermore, many authors [10,12,13,31,32] studied weighted L^p-regularity theory for various kinds of second-order elliptic linear and nonlinear problems. Moreover, Byun, Ok, Palagachev and Softova [11] obtained the global gradient estimates in weighted Orlicz spaces for weak solutions of parabolic systems in divergence form with bounded measurable coefficients when the associated nonhomogeneous term belongs to weighted Orlicz spaces. Recently, Byun and Lee [9] proved that

$$|f|^2\in L^{\phi}_w(\Omega_T)\Rightarrow |u|^2,|Du|^2,|D^2u|^2,\ |u_t|^2\in L^{\phi}_w(\Omega_T)$$

for (1.1)–(1.2). The purpose of this paper is to extend the result in [9] to the following result under the same assumptions

$$f\in L^{\phi}_w(\Omega_T)\Rightarrow u,Du,D^2u,\ u_t\in L^{\phi}_w(\Omega_T).$$

We remark that our approach and the main tools are similar to those in [9]. In a sense, this paper is just a comment on Byun and Lee's paper [9].

Since the 1960s, the need to use wider spaces of functions than Sobolev spaces came from various practical problems. Orlicz spaces have been studied as the generalization of Sobolev spaces since they were introduced by Orlicz [34] (see [3,4,19–21,27]). The theory of Orlicz spaces plays a crucial role in many fields of mathematics including geometric, probability, stochastic, Fourier analysis and PDE (see [35]).

We denote Φ by

$$\Phi=\left\{\phi:[0,+\infty)\longrightarrow[0,+\infty)\ \middle|\ \phi \text{ is increasing and convex}\right\}. \tag{1.4}$$

Definition 1.2. A function $\phi\in\Phi$ is said to be a Young function if

$$\lim_{t\to 0+}\frac{\phi(t)}{t}=\lim_{t\to+\infty}\frac{t}{\phi(t)}=0.$$

Definition 1.3. A Young function $\phi\in\Phi$ is said to satisfy the global Δ_2 condition, denoted by $\phi\in\Delta_2$, if there exists a positive constant K such that for every $t>0$,

$$\phi(2t)\leq K\phi(t).$$

Moreover, a Young function $\phi\in\Phi$ is said to satisfy the global ∇_2 condition, denoted by $\phi\in\nabla_2$, if there exists a number $a>1$ such that for every $t>0$,

$$\phi(t)\leq\frac{\phi(at)}{2a}.$$