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On the free boundary problem for the Oldroyd-B Model in the maximal L_p–L_q regularity class

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ABSTRACT

In the present work, we prove the local well-posedness of non-Newtonian compressible viscous barotropic fluid flow of Oldroyd-B type with free surface in a bounded domain of N-dimensional Euclidean space ($N \geq 2$). The key step is to prove the maximal L_p–L_q regularity theorem for the linearized equation with the help of the $\mathcal{R}$-bounded solution operators for the corresponding resolvent problem and Weis's operator valued Fourier multiplier theorem.

1. Introduction and main result

Let Ω be a bounded domain in the N-dimensional Euclidean space $\mathbb{R}^N$ ($N \geq 2$) whose boundary consists of two parts Γ_0 and Γ_1, where $\Gamma_0 \cap \Gamma_1 = \emptyset$. The Ω is occupied by a compressible viscous barotropic non-Newtonian fluid of Oldroyd-B type. The present paper deals with the problem of determining the region $\Omega_t \subset \mathbb{R}^N$, the density field $\rho = \rho(x,t)$, the elastic tensor $\tau = \tau(x,t)$, and the velocity field $\mathbf{u} = (u_1(x,t), \ldots, u_N(x,t))$, which satisfy the system of equations:

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho \mathbf{u}) = 0 & \text{in } \Omega_t, \\ \rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) - \operatorname{Div} \mathbf{T}(\mathbf{u}, P(\rho)) = \beta \operatorname{Div} \tau & \text{in } \Omega_t, \\ \partial_t \tau + \mathbf{u} \cdot \nabla \tau + \gamma \tau = \delta \mathbf{D}(\mathbf{u}) + g_\alpha(\nabla \mathbf{u}, \tau) & \text{in } \Omega_t, \\ (\mathbf{T}(\mathbf{u}, P(\rho)) + \beta \tau)\mathbf{n}_t = -P(\rho_*)\mathbf{n}_t & \text{on } \Gamma_t, \\ \mathbf{u} = 0 & \text{on } \Gamma_0, \\ (\rho, \mathbf{u}, \tau)|_{t=0} = (\rho_* + \theta_0, \mathbf{u}_0, \tau_0) & \text{in } \Omega, \\ \Omega_t|_{t=0} = \Omega_0, \quad \Gamma_t|_{t=0} = \Gamma_1 \end{cases} \tag{1.1}$$

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for $0 < t < T$. Here, ρ_* is a positive constant describing the mass density of the reference domain Ω, $\mathbf{T}(\mathbf{u}, P(\rho))$ the stress tensor of the form

$$\mathbf{T}(\mathbf{u}, \rho) = \mathbf{S}(\mathbf{u}) - P(\rho)\mathbf{I} \quad \text{with } \mathbf{S}(\mathbf{u}) = \mu\mathbf{D}(\mathbf{u}) + (\nu - \mu)\mathrm{div}\,\mathbf{u}\mathbf{I}, \tag{1.2}$$

$\mathbf{D}(\mathbf{u})$ the doubled deformation tensor whose (i, j) components are $D_{ij}(\mathbf{u}) = \partial_i u_j + \partial_j u_i$ $(\partial_i = \partial/\partial x_j)$, $\mathbf{I}$ the $N \times N$ identity matrix, μ, ν, β, γ and δ are positive constants (μ and ν are the first and second viscosity coefficients, respectively), $\mathbf{n}_t$ is the unit outer normal to Γ_t, $P(\rho)$ a C^∞ function defined for $\rho > 0$ which satisfies that $P'(\rho) > 0$ for $\rho > 0$. Moreover, the function $g_\alpha(\nabla\mathbf{u}, \tau)$ has a form

$$g_\alpha(\nabla\mathbf{u}, \tau) = \mathbf{W}(\mathbf{u})\tau - \tau\mathbf{W}(\mathbf{u}) + \alpha(\tau\mathbf{D}(\mathbf{u}) + \mathbf{D}(\mathbf{u})\tau), \tag{1.3}$$

where α is a constant with $-1 \le \alpha \le 1$ and $\mathbf{W}(\mathbf{u})$ the doubled antisymmetric part of the gradient $\nabla\mathbf{u}$ whose (i, j) components are $W_{ij}(\mathbf{u}) = \partial_i u_j - \partial_j u_i$. Finally, for any matrix field $\mathbf{K}$ whose components are K_{ij}, the quantity $\mathrm{Div}\,\mathbf{K}$ is an N vector whose ith component is $\sum_{j=1}^N \partial_j K_{ij}$, and also for any vector of functions $\mathbf{u} = (u_1, \ldots, u_N)$, $\mathrm{div}\,\mathbf{u} = \sum_{j=1}^N \partial_j u_j$, and $\mathbf{u} \cdot \nabla\mathbf{u}$ is an N vector whose ith component is $\sum_{j=1}^N u_j\partial_j u_i$. We assume that the boundary of Ω_t consists of Γ_0 and Γ_t with $\Gamma_0 \cap \Gamma_t = \emptyset$.

Aside from the dynamical system (1.1), a further kinematic condition for Γ_t is satisfied, which gives

$$\Gamma_t = \{x \in \mathbb{R}^N \mid x = \mathbf{x}(\xi, t) \ (\xi \in \Gamma_1)\}, \tag{1.4}$$

where $\mathbf{x} = \mathbf{x}(\xi, t)$ is the solution to the Cauchy problem:

$$\Gamma_t = \{x \in \mathbb{R}^N \mid x = \mathbf{x}(\xi, t) \ (\xi \in \Gamma_1)\}. \tag{1.5}$$

Concerning the free boundary problem of the viscous compressible barotropic Newtonian fluid flow, the local well-posedness and global well-posedness have been studied in the L_2 Sobolev–Slobodetskii space by Denisova and Solonnikov [4,3], Secchi and Valli [17–19], Solonnikov and Tani [28,30,31], and Zajaczkowski [34,35], and in the L_p–L_q maximal regularity class by Shibata et al. [7,24]. Recently, M. Nesensohn [14] proved the local well-posedness of the free boundary problem for the non-Newtonian fluid flow of Oldroyd-B type in the incompressible viscous fluid case (further references are found in [14]). On the other hand, Shi, Wang and Zhang [20] investigated the asymptotic stability for 1-dimensional motion of non-Newtonian compressible fluids using L_2 energy method. Meanwhile, global existence of strong solutions of Navier–Stokes equations with non-Newtonian potential for 1-dimensional isentropic compressible fluids has been studied by Liu, Yuan and Lie [9]. The purpose of this paper is to study the local well-posedness of problem (1.1).

To prove the local well-posedness of problem (1.1), we use the Lagrangian coordinate in order to transform the time dependent domain Ω_t to the fixed domain Ω. Let $\mathbf{u}(x, t)$ and $\mathbf{v}(\xi, t)$ be velocity fields in the Euler coordinate and in the Lagrangian coordinate, respectively. The Euler coordinate system $\{x\}$ and Lagrangian coordinate system $\{\xi\}$ are connected by the relation:

$$x = \xi + \int_0^t \mathbf{v}(\xi, s)ds \equiv \mathbf{X}_\mathbf{v}(\xi, t),$$

where, $\mathbf{v}(\xi, t) = (v_1(\xi, t), \ldots, v_N(\xi, t)) = \mathbf{u}(\mathbf{X}_\mathbf{v}(\xi, t), t)$. Let A be the Jacobi matrix of the transformation $x = \mathbf{X}_\mathbf{v}(\xi, t)$, whose (i, j) element is $a_{ij} = \delta_{ij} + \int_0^t (\frac{\partial v_i}{\partial \xi_j})(\xi, s)ds$. There exists a small number σ such that if

$$\max_{i,j=1,\ldots,N} \left\| \int_0^t \frac{\partial v_i}{\partial \xi_j}(\cdot, s)ds \right\|_{L_\infty(\Omega)} < \sigma \quad (0 < t < T), \tag{1.6}$$

then A is invertible, that is, $\det A \neq 0$. Thus, we have $\nabla_x = A^{-1}\nabla_\xi = (\mathbf{I} + \mathbf{V}_0(\int_0^t \nabla\mathbf{v}(\xi, s)ds))\nabla_\xi$, where $\mathbf{V}_0(\mathbf{K})$ is an $N \times N$ matrix of C^∞ functions with respect to $\mathbf{K} = (k_{ij})$ for $|\mathbf{K}| < 2\sigma$ and $\mathbf{V}_0(0) = 0$. Here and hereafter, k_{ij} denote corresponding variables to $\int_0^t (\frac{\partial v_i}{\partial \xi_j})(\cdot, s)ds$. Let $\mathbf{n}$ be the unit outward normal to