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Regularity on the interior for some class of nonlinear second-order elliptic systems

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A B S T R A C T

The interior $C^{1,\gamma}$- regularity is proved for weak solutions to a class of nonlinear second-order elliptic systems. It is typical for the system belonging to the class that the continuity moduli of the gradients of its coefficients become slow growing sufficiently far from zero. This property guarantees the regularity of the gradients of solutions to such system in a case when the ellipticity constant is big enough. Some characteristic features of the obtained result are illustrated by examples at the end of the paper.

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1. Introduction

In this paper, we give conditions guaranteeing that a weak solution to a nonlinear elliptic system

$$-D_\alpha\left(A_i^\alpha(Du)\right)=0 \quad \text{in } \Omega,\ i=1,\dots,N \tag{1}$$

belongs to $C^{1,\gamma}_{\text{loc}}(\Omega,\mathbb{R}^N)$ space. Throughout the whole text, we use the summation convention over repeated indices.

By the weak solution to the system (1) we understand $u\in W^{1,2}_{loc}(\Omega,\mathbb{R}^N)$ such that

$$\int_\Omega A_i^\alpha(Du)D_\alpha\varphi^i\,dx=0,\quad \forall\,\varphi\in W_0^{1,2}(\Omega,\mathbb{R}^N).$$

Here $\Omega\subset\mathbb{R}^n$ is a bounded open set, $n\geq 3$ and continuously differentiable coefficients $(A_i^\alpha)_{i=1,\dots,N,\alpha=1,\dots,n}$ have the linear controlled growth and satisfy the strong uniform ellipticity condition. More precisely,

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denoting

$$A_{ij}^{\alpha\beta}(p) = \frac{\partial A_i^\alpha}{\partial p_j^\beta}(p)$$

and supposing $A_i^\alpha(0) = 0$, we require as follows.

(i) There exists a constant $M > 0$ such that for every $p \in \mathbb{R}^{nN}$

$$\sum_{i,\alpha} |A_i^\alpha(p)| \le M(1 + |p|).$$

(ii)

$$\sum_{i,j,\alpha,\beta} \left|A_{ij}^{\alpha\beta}(p)\right| \le M.$$

(iii) The strong ellipticity: there exists $\nu > 0$ such that for every $p, \xi \in \mathbb{R}^{nN}$

$$A_{ij}^{\alpha\beta}(p)\xi_\alpha^i\xi_\beta^j \ge \nu|\xi|^2.$$

(iv) There exists a real function ω defined and continuous on $[0, \infty)$, which is bounded, nondecreasing, increasing on a neighbourhood of zero, $\omega(0) = 0$ and such that for all $p, q \in \mathbb{R}^{nN}$

$$\sum_{i,j,\alpha,\beta} \left|A_{ij}^{\alpha\beta}(p) - A_{ij}^{\alpha\beta}(q)\right| \le \omega(|p - q|).$$

We set $\omega_\infty = \lim_{t\to\infty} \omega(t) \le 2M$.

Here it is worth to point out (see [8, p. 169]) that for uniformly continuous coefficients $A_{ij}^{\alpha\beta}$ there exists a real function ω satisfying the assumption (iv) and, vice versa, (iv) implies the uniform continuity of coefficients and absolute continuity of ω on $[0, \infty)$.

In this paper we will consider the continuous function ω given in the form

$$\omega(t) = \begin{cases} \dfrac{(1+\beta)^\beta\sqrt{\varepsilon}}{\left(1 + \ln\frac{t_o e^\beta}{t}\right)^\beta} & \text{for } 0 < t < t_o,\ \beta > 0, \\ \omega_\infty\sqrt{\ln\left(1 + \dfrac{e^{\varepsilon/\omega_\infty^2} - 1}{t_0}t\right)}, & \text{for } t_o \le t \le t_1, \\ \omega_\infty & \text{for } t > t_1 \end{cases} \tag{2}$$

where the constant $\varepsilon > 0$ will be specified below. Here we only mention that the first part in the definition of the function ω can be substituted by whichever function in such a way that ω will fulfil (iv).

It is well known that the Dirichlet problem

$$\begin{aligned} -D_\alpha(A_i^\alpha(Du)) = 0 \quad &\text{in } \Omega,\ i = 1, \dots, N, \\ u = g \quad &\text{on } \partial\Omega \end{aligned} \tag{3}$$

has a unique solution $u \in W^{1,2}(\Omega, \mathbb{R}^N)$. Moreover, for boundary function $g \in W^{1,2}(\Omega, \mathbb{R}^N)$ it holds

$$\int_\Omega |Du - (Du)_\Omega|^2\, dx \le \left(\frac{M}{\nu}\right)^2 \int_\Omega |Dg - (Dg)_\Omega|^2\, dx \tag{4}$$

where $(v)_\Omega$ means the integral mean value of a function v over Ω. The foregoing estimate can be proved by a standard technique (see [4, Appendix]).

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