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On the asymptotic formulas of solutions to the boundary value problem without initial condition for Schrödinger systems in domain with conical points

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ABSTRACT

In this paper, boundary value problems without initial condition for Schrödinger systems in domains with conical points are considered. We establish several results on the asymptotic behavior of solutions near singular points.

0. Introduction

There exist a large number of investigations of boundary value problems in non-smooth domains with conical points. At present, the theory of boundary value problems for elliptic equations and systems in such domains has been thoroughly investigated (see, e.g., [10–12,17,18]). Parallel with this theory, boundary value problems for non-stationary equations and systems have been considered by many authors (see [2,5,7,9] for example).

In this paper, we are concerned with Schrödinger systems in cylinders with conical points. This topic has been studied in many works with different approaches. In [15,16], J-L. Lions, F. Magenes considered Schrödinger equations whose coefficients are independent of the time variable. In [3,4], the first initial

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boundary value problem for this kind of equation with coefficients depending on both time and spatial variables has been investigated. In these works, to consider the asymptotic behavior of solutions in a neighborhood of conical points, the authors just studied the case when the associated spectrum problem has simple eigenvalues or has semisimple eigenvalues having invariant multiplicities. Different from the above-mentioned papers, we consider in the present work the asymptotics of solutions to the boundary value problem without initial condition for Schrödinger systems in domains with conical points. We weaken the assumptions on the eigenvalues of the associated spectrum problem (see condition **(H)** in Section 2 for a precise statement) but we still obtain asymptotic formulas of solutions.

The paper is organized as follows. In Section 1, we present some notation and the formulation of the problem. The main results, Theorem 2.3, is stated in Section 2. The proof of the main theorem is given in Section 3. Finally, in the last section we discuss an example.

1. Notation and formulation of the problem

Let Ω be a bounded domain in $\mathbb{R}^n$ $(n \geq 2)$ with the boundary $S = \partial\Omega$. Assume that S is an infinitely differentiable surface everywhere, except for the coordinate origin and in a neighborhood U_0 of the origin, $\Omega \cap U_0$ coincides with the cone $K = \{x : x/|x| \in G\}$, where G is a smooth domain on the unit sphere S^{n-1}. We use Q to refer to $\Omega \times \mathbb{R}$, Γ to refer to $S \times \mathbb{R}$ and K_∞ to refer to $K \times \mathbb{R}$. For each multi-index $\alpha = (\alpha_1, \dots, \alpha_n) \in \mathbb{N}^n$, set $|\alpha| = \alpha_1 + \dots + \alpha_n$ and $D^\alpha = D^\alpha_x = \partial^{|\alpha|}/\partial^{\alpha_1}_{x_1} \dots \partial^{\alpha_n}_{x_n}$.

Denote $u(x,t) = (u_1(x,t), \dots, u_s(x,t))$, $D^\alpha u = (D^\alpha u_1, \dots, D^\alpha u_s)$, $|D^\alpha u|^2 = \sum_{i=1}^s |D^\alpha u_i|^2$ and $u_{t^j} = (\frac{\partial^j u_1}{\partial t^j}, \dots, \frac{\partial^j u_s}{\partial t^j})$, $|u_{t^j}|^2 = \sum_{i=1}^s |\frac{\partial^j u_i}{\partial t^j}|^2$.

Let us introduce some functional spaces (see [6]) used in this paper.

We use $H^k(\Omega)$ to be the space of s-dimensional vector functions defined in Ω with the norm

$$\|u\|_{H^k(\Omega)} = \Big(\sum_{|\alpha|=0}^{k} \int_\Omega |D^\alpha u|^2 dx \Big)^{\frac{1}{2}},$$

and $H^{k-1/2}(S)$ to be the space of traces of vector functions from $H^k(\Omega)$ on S with the norm

$$\|u\|_{H^{k-1/2}(S)} = \inf\{\|w\|_{H^k(\Omega)} : w \in H^k(\Omega),\ w|_S = v\}.$$

Denote by $H^{k,l}(Q)$ the space consisting of all vector functions $u : Q \longrightarrow \mathbb{C}^s$ satisfying

$$\|u\|_{H^{k,l}(Q)} = \Big(\int_Q \Big(\sum_{|\alpha|=0}^{k} |D^\alpha u|^2 + \sum_{j=1}^{l} |u_{t^j}|^2 \Big) dxdt \Big)^{\frac{1}{2}} < +\infty,$$

and $H^{k,l}(-\gamma, Q)$ is the space of vector functions with the norm

$$\|u\|_{H^{k,l}(-\gamma,Q)} = \Big(\int_Q \Big(\sum_{|\alpha|=0}^{k} |D^\alpha u|^2 + \sum_{j=1}^{l} |u_{t^j}|^2 \Big) e^{-2\gamma t} dxdt \Big)^{\frac{1}{2}}.$$

In particular,

$$\|u\|_{H^{k,0}(-\gamma,Q)} = \Big(\sum_{|\alpha|=0}^{k} \int_Q |D^\alpha u|^2 e^{-2\gamma t} dxdt \Big)^{\frac{1}{2}}.$$

Especially, we set $L_2(-\gamma, Q) = H^{0,0}(-\gamma, Q)$.

We define $H^{l,k}_\beta(-\gamma, Q)$ as the space of all functions $u(x,t)$ which have generalized derivatives $D^\alpha u$, u_{t^j}, $|\alpha| \leq l$, $1 \leq j \leq k$ satisfying

$$\|u\|^2_{H^{l,k}_\beta(-\gamma,Q)} = \int_Q \Bigg(\sum_{|\alpha|=0}^{l} r^{2(\beta+|\alpha|-l)} |D^\alpha u|^2 + \sum_{j=1}^{k} |u_{t^j}|^2 \Bigg) e^{-2\gamma t} dxdt < \infty.$$