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Asymptotic properties of multifunctions, families of measures and Markov operators associated with cocycles

Grzegorz Guzik

AGH Science and Technology University, Faculty of Applied Mathematics, Al. Mickiewicza 30, 30-059 Krakow, Poland

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ABSTRACT

We consider asymptotic properties of families of multifunctions generated by cocycles, families of measures as well as Markov operators associated with such a mapping. The results we apply to random dynamical systems show connection of obtained attractors for families of multifunctions with global set attractors and also connection of supports of attracting measures for Markov families with global point attractors. The use of topological limits instead of standard Hausdorff distance lets us to get attractors under some quite general assumptions only on a cocycle mapping but without any assumptions on so-called parameter space and without standard assumption on existence of a compact absorbing set.

1. Introduction

The notion of attracting sets is one of the basic concepts in the theory of dynamical systems. Such a set attracts, in some sense, sets from desired class of subsets of so-called phase space and determines the long-term behavior of dynamical system. Attractors of classical autonomous dynamical system are investigated since many years, and the criteria of existence, the form and properties of different types of attractors of nonautonomous as well as random dynamical systems are intensively studied during last two decades (see for example [2,9–12,21] for details and the references therein).

In the theory of nonautonomous/random dynamical system the notion of cocycle mapping is fundamental (see [1,21]). A cocycle is, roughly speaking, a mapping which acts on the product of a parameter space and a phase space (usually of different nature), inducing an autonomous skew product (semi)flow. On a

E-mail address: guzik@agh.edu.pl.

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parameter space an autonomous (semi)flow is given which can be, in general, interpreted as perturbations or a noise. The natural areas when cocycle mappings appear are nonautonomous/stochastic difference and differential equations (see for example [1,8,21] for numerous examples and the references therein) and also the theory of control systems (see [6,7,14] and the cited bibliography therein). Note just now that there are examples of cocycle mappings which could be obtained out of context of difference and differential nonautonomous/stochastic equations (see for example [16,17]).

The paper divides into three main parts. First (Section 4) we consider general cocycle mappings and our goal is to show some sufficient conditions for existence of an attractor of a family of multifunctions induced by a given cocycle. Such a (deterministic) set attracts globally all bounded subsets of the phase space which is supposed to be an arbitrary metric space. As it follows from Proposition 4.3 (see also discussion in [11, Remark 2.4(ii)]) the obtained attractor coincides with the global random (set) attractor (in the random case, cf. also [9,10,12]) and the global forward/pullback attractor (in the nonautonomous case, cf. also [2,21], see also Remark 4.2). Obtained in such a way attractor need not to be compact, however it can be the union of compact components (clearly it is always a closed set). There is also another reason why we consider such a type of attractors. It is remarkable that attractors (fractals) of iterated function systems (IFS-s) are always noticed as a single set rather then a family of sets. If one would like to consider classical IFS consisting of some contractions our approach is concurrent with that classical of IFS-s by Barnsley and Hutchinson (see the vast bibliography starting from early [18], classical [3] and numerous papers and books of successors up to now, especially [27]). It was shown (see for example [16, Example 3.1]) that any iterated function system can be described as a discrete cocycle.

We show that under quite general assumptions only on cocycle mappings we obtain attractors. Note for sure that we do not need *any* structure on the parameter space. Most of similar results where existence of global (in some sense) attractors were considered could be obtained under many assumptions on the parameter space (measurability, in the random case; metrizability or even compactness, in the nonautonomous case), on the phase space (separability, completeness etc.), as well as on the cocycle mapping itself (continuity, differentiability etc.). Note that there are results of that sort where none is assumed on the parameter space, but then the phase space need to be selected very carefully [29]. Our results could be applied for both nonautonomous and random dynamical systems. There is another profit of our approach: most of classical results on existence of global set attractors were obtained under assumption of existence of so-called compact absorbing set or compact dissipativity of the system (see [5,21]), and in applications one who try to obtain attractors brings laborious job of finding absorbing sets. On the other hand there are examples when attractor exists (see simple Example 4.7), but there is no absorbing compact set or even a family of such sets. We obtain attractors without those assumptions. Moreover, we can see that existence of attractor could be independent of a driving dynamical system or perturbations influencing to the system.

The second part (Section 5) is devoted to asymptotic properties of families (nets) of Markov operators acting on the space of finite Borel measures of a Polish space and families of supports of measures. In particular it is proved that for asymptotically stable family of Markov–Feller operators associated family of Markov multifunctions admits so-called semiattractor which contains or even simply is a support of the attracting probability measure. Our results extend those contained in papers [24–27] by A. Lasota and J. Myjak concerning discrete semigroups generated by a single Markov operator. They can be applied also to semigroups of Markov operators.

In the last part (Section 6) we apply obtained theorems to random dynamical systems. We consider the family of Markov–Feller operators induced by an arbitrary random dynamical system as well as white noise one. It is shown that a support of attracting measure for such a family of operators is strictly connected with a minimal global point attractor.

The main tool we use are topological limits. In modern theory of multifunctions this point of view seems to be more widely applied (see for example [4]). Note that such a point of view is not only more general and