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Generalised morphological image diffusion

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ABSTRACT

Relationships between linear and morphological scale-spaces have been considered by various previous works. The aim of this paper is to study how to generalise the discrete and continuous diffusion-based approaches in order to introduce nonlinear filters whose limit effects mimic the asymmetric behaviour of morphological dilation and erosion, as well as other multiscale filters, hybrid between the standard linear and morphological filters. A methodology based on the counter-harmonic mean is adopted here. Partial differential equations are formulated and details of numerical implementation are discussed to illustrate the various studied cases: isotropic, edge-preserving and coherence-enhancing diffusion. We also found a new way to derive the classical link between Gaussian scale-space and dilation/erosion scale-spaces based on quadratic structuring functions in the discrete and continuous setting. We have included some preliminary applications of the generalised morphological diffusion to solve image processing problems such as denoising and image enhancement in the case of asymmetric bright/dark image properties.

1. Introduction

Two fundamental paradigms of image filtering appear distinguished nowadays in the state-of-the-art. On the one hand, differential methods inspired from the (parabolic) heat equation, including isotropic diffusion [32], nonlinear diffusion [41,11], anisotropic diffusion [54], etc. The main properties of these techniques are the appropriateness to deal with the notion of scale-space of image structures and the ability to process symmetrically the bright/dark image structures. Practical algorithms involve (local-adaptive) kernel convolution as well as PDE-formulation and subsequent numerical solutions. The interested reader should refer to basic Refs. [21,16,53]. On the other hand, mathematical morphology operators [45,47] are formulated in terms of geometric notions as well as in terms of complete lattice theory. Morphological filters entail mainly the computation of supremum and infimum values in neighbourhoods (or structuring elements) which correspond respectively to the dilation and the erosion, the two basic operators. Morphological operators present also good scale-space properties [28,51,44], but by the natural duality of complete lattices,

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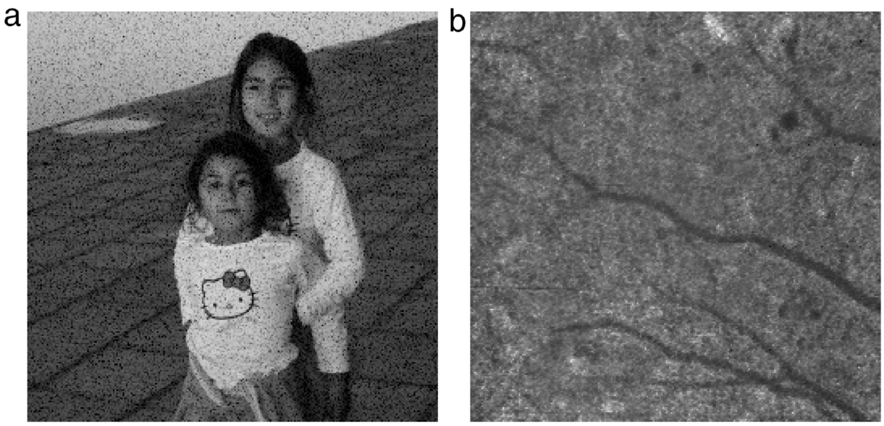

Fig. 1. Two image examples presenting asymmetric bright/dark image structures. These images will be studied in Section 7.

most operators appear by pairs and one acts on bright structures and the other one on dark structures. This latter property of asymmetry is in fact an advantage which allows defining evolved operators by composition product of a pair of dual ones. For instance, the opening (resp. closing) is obtained by the product of an erosion (resp. dilation) followed by a dilation (resp. erosion), then the product of openings and closings leads to the alternate filters and other families of morphological filters [45,47]. Diffusion involves blurring image structures whereas morphological dilation and erosion involve enhancement of image structure transitions. In fact, morphological operators are related to geometric optics models and in particular to the (hyperbolic) eikonal equation. Hence, there exists also a well motivated formulation of morphological operators using PDEs [1,4,7,37]. This differential or continuous-scale morphology can be solved using numerical algorithms for curve evolution [43]. Thus multiscale flat dilation/erosion by disks as structuring elements (resp. unflat dilation/erosion by parabolic structuring functions) can be modelled in a continuous framework. Morphological operators using geometric structuring elements are today one of the most successful areas in image processing. However, it is obvious that the soundness and maturity of numerical methods to implement the different versions of image diffusion constitute an advantage against continuous morphology implementation, which requires more specific numerical schemes to achieve robust results [49,6].

Motivation and Aim: Asymmetrisation of diffusion image filtering. Let us consider the two images given in Fig. 1. The first image is an example of impulse-noise corrupted image, where the noise presents a mean value which is significatively smaller than the mean image value. In other words, the noise is asymmetrically shifted towards dark values. A successful denoising approach, based for instance on nonlinear adaptive diffusion, should be able to deal with this asymmetry. The second image is a low SNR retinal image and one of its typical applications involves to extract the vessels. The latter structures are dark with respect to their background. Vessel detection can be solved using morphological operators, but the result would be poor without a prior enhancement of the image using typically a coherence-enhancing diffusion-based filtering step. Obviously, the relation of intensities between the vessels and the background could be of help to adapt the nonlinearity of the appropriate anisotropic diffusion.

In this context, the aim of this paper is to study how to generalise the diffusion-based approaches in order to introduce a family of nonlinear filters whose effects mimic the asymmetric behaviour of morphological dilation and erosion, as well as other evolved adaptive filters for edge-preserving and coherence-enhancement. A parameter P of nonlinearity is used to tune the asymmetrisation (according to the sign of P) and limit effects (according to the modulus of P). Our goal is therefore to propose a new approach of generalised