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Optimality and duality for robust multiobjective optimization problems☆

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ABSTRACT

This paper deals with a *robust multiobjective* optimization problem involving *nonsmooth/nonconvex* real-valued functions. We establish necessary/sufficient optimality conditions for robust (weakly) Pareto solutions of the considered problem. These optimality conditions are presented in terms of multipliers and limiting subdifferentials of the related functions. In addition, we address a dual (robust) multiobjective problem to the primal one, and explore weak/strong duality relations between them under assumptions of (strictly) generalized convexity.

1. Introduction

Due to prediction errors or lack of information, the data of a real-world optimization problem are often *uncertain*; i.e., they are not known precisely when the problem is solved [2]. *Robust optimization* has emerged as a remarkable deterministic framework for studying mathematical programming problems with uncertain data. Theoretical and applied aspects in the area of robust optimization have been studied intensively by many researchers; see e.g., [1–6,8,11,14,19,20,23] and the references therein.

For each $i \in \{1, \ldots, l\}$, let Ω_i be a nonempty compact subset of $\mathbb{R}^{n_i}, n_i \in \mathbb{N} := \{1, 2, \ldots\}$. Let $f := (f_1, f_2, \ldots, f_m)$ be a vector function with locally Lipschitz components defined on $\mathbb{R}^n$, and let $\mathbb{R}^m_+$ be the nonnegative orthant of $\mathbb{R}^m$.

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We consider an *uncertain multiobjective* optimization problem of the form:

$$\min_{\mathbb{R}^m_+} \big\{f(x) \mid g_i(x,\omega_i) \le 0,\ i=1,\dots,l\big\}, \tag{UP}$$

where $x \in \mathbb{R}^n$ is the vector of decision variables, $\omega_i \in \Omega_i, i=1,\dots,l$, are *uncertain* parameters, and $g_i : \mathbb{R}^n \times \Omega_i \to \mathbb{R}, i=1,\dots,l$, are given functions. Here, *uncertainty* can be interpreted in a manner that the parameter vectors $\omega_i, i=1,\dots,l$, are not known exactly at the time of the decision.

For investigating problem (UP), one usually associates with it the so-called *robust* counterpart:

$$\min_{\mathbb{R}^m_+} \big\{f(x) \mid x \in C\big\}, \tag{RP}$$

where the feasible set C is defined by

$$C := \big\{x \in \mathbb{R}^n \mid\ g_i(x,\omega_i) \le 0\ \forall \omega_i \in \Omega_i,\ i=1,\dots,l\big\}. \tag{1.1}$$

It is worth observing here that (see e.g., [12, Theorem 3.1]) if the functions $g_i(\cdot,\omega_i), \omega_i \in \Omega_i,\ i=1,\dots,l$ are convex, then the problem (RP) is feasible (i.e., $C \neq \emptyset$) if and only if

$$(0,-1) \notin \operatorname{cl}\operatorname{cone}\big\{\cup \operatorname{epi} g_i^*(\cdot,\omega_i) \mid \omega_i \in \Omega_i,\ i=1,\dots,l\big\}, \tag{1.2}$$

where $\operatorname{cl}\Omega$ and $\operatorname{cone}\Omega$ denote the closure and convex conical hull of $\Omega \subset \mathbb{R}^{n+1}$ respectively, while $\operatorname{epi} g_i^*$ stands for the epigraph of the conjugate function of g_i.

The following concepts of solutions can be found in the literature; see e.g., [13,21].

Definition 1.1. (i) We say that a vector $\bar{x} \in \mathbb{R}^n$ is a *local robust Pareto solution* of problem (UP), and write $\bar{x} \in loc\mathcal{S}(RP)$, if $\bar{x}$ is a local Pareto solution of problem (RP), i.e., $\bar{x} \in C$ and there is a neighborhood U of $\bar{x}$ such that

$$\forall x \in C \cap U, \quad f(x) - f(\bar{x}) \notin -\mathbb{R}^m_+ \setminus \{0\}.$$

(ii) A vector $\bar{x} \in \mathbb{R}^n$ is called a *local robust weakly Pareto solution* of problem (UP), and write $\bar{x} \in loc\mathcal{S}^w(RP)$, if $\bar{x}$ is a local weakly Pareto solution of problem (RP), i.e., $\bar{x} \in C$ and there exists a neighborhood U of $\bar{x}$ such that

$$\forall x \in C \cap U, \quad f(x) - f(\bar{x}) \notin -\operatorname{int}\mathbb{R}^m_+,$$

where $\operatorname{int}\mathbb{R}^m_+$ signifies the topological interior of $\mathbb{R}^m_+$.

In the above definitions, if $U = \mathbb{R}^n$, then one has the concepts of *robust Pareto solution* and *robust weakly Pareto solution* for problem (UP), and in this case we denote these solution sets by $\mathcal{S}(RP)$ and $\mathcal{S}^w(RP)$, respectively.

It is worthy of mention here that a local *robust* (weakly) Pareto solution of problem (UP), i.e., a local (weakly) Pareto solution of (RP), is recognized as a *best resisted uncertainty* (*hard*) solution of problem (UP). Roughly speaking, it is a solution of (UP) that remains feasible for every perturbed counterpart of the constraints, provided that the perturbations belong to prescribed regions. We refer the reader to the book [2] for more details on specific formulations, and discussions about (scalar) robust optimization.

To the best of our knowledge, there is no work has been published dealing with optimality conditions and duality in the above-defined optimization problems involving *nonsmooth/nonconvex* functions. Our main purpose in this paper is to establish necessary/sufficient optimality conditions for *local robust (weakly) Pareto solutions* of problem (UP) formulated by nondifferentiable/nonconvex functions. These optimality conditions are presented in terms of multipliers and limiting subdifferentials of the related functions. Along with