



Systems of partial differential equations in porous medium



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ABSTRACT

We investigate systems of degenerate parabolic equations idealizing reactive solute transport in porous media. Taking advantage of the inherent structure of the system that allows to deduce a scalar Generalized Porous Medium Equation for the sum of the solute concentrations, we show existence of a unique weak solution to the coupled system and derive regularity estimates. We also prove that the system supports solutions propagating with finite speed thus giving rise to free boundaries and interaction of compactly supported initial concentrations of different species.

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1. Introduction

1.1. Physical background

The transport of pollutants in subsurface environments is a complex phenomenon modeled by advection–diffusion–reaction equations describing the evolution of contaminant concentrations in porous medium through various mechanical and chemical processes. In what follows, we will briefly describe the equations governing the transport of one or more contaminants through a fluid-saturated porous medium, i.e. a medium characterized by a partitioning of its total volume into a solid phase (solid matrix) and a void or pore space that is filled by one or more fluids. For a more detailed description, we refer to [5].

Let $\Omega \subset \mathbb{R}^n$ be a domain occupied by a porous medium, let $z = z(x, t)$ denote the solute concentration of one contaminant component in the fluid phase and assume that the flow is at steady state and that the transport is described by advection, molecular diffusion, mechanical dispersion and chemical reaction (adsorption) between a solute and the surrounding porous skeleton. More than often the adsorption,

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accumulation of a pollutant on the solid matrix at the fluid–solid interface, is in fact the main mechanism responsible for the contaminant transport in soil. Based on a continuum approach, the mass conservation for z can be written as

$$\phi \frac{\partial z}{\partial t} + \rho \frac{\partial b_a}{\partial t} + \nabla \cdot (z\mathbf{V} - \phi \mathbf{D}\nabla z) = f, \quad (1.1)$$

where $\phi \in (0, 1)$ is the constant porosity, $z\mathbf{V}$ denotes the advective water flux (\mathbf{V} is the Darcy velocity), $\rho > 0$ is the constant bulk density of the solid matrix, \mathbf{D} stands for the hydrodynamic dispersion matrix describing both the molecular diffusion and the mechanical dispersion between the solute and the surrounding porous medium, and the source or sink terms are denoted by f . Moreover, an *isotherm* $b_a = b_a(z)$ describes the concentration of contaminant adsorbed on the solid matrix through a reactive adsorption process at constant temperature, which is assumed here to be infinitely fast (equilibrium). The most commonly used nonlinear equilibrium isotherms [5,27] are

$$\begin{aligned} b_a(z) &= K z^p && \text{Freundlich,} \\ b_a(z) &= K \frac{z}{1+z} && \text{Langmuir,} \end{aligned}$$

where $K > 0$ is constant and the Freundlich exponent p is generally chosen in $(0, 1)$ (the smaller the p , the higher the adsorption at low concentrations).

The values $p \in (0, 1)$ make (1.1) singular at $z = 0$ because, at least formally, $\partial_t b_a(z) = b'_a(z)\partial_t z$ and $b'_a(0) = \infty$. The equation may thus exhibit finite speed of propagation of compactly supported initial solutions giving rise to free boundaries that separate the region where the solute concentration vanishes from that with positive concentration. This is in marked contrast with the behavior of solutions for Freundlich exponents $p \geq 1$ or Langmuir isotherms, since the equation becomes nonsingular and the information propagates with infinite speed as usual for uniformly parabolic equations.

Eq. (1.1), complemented with suitable initial and boundary conditions, and all its variants arising from different equilibrium and non-equilibrium, linear or non-linear, isotherms, have attracted considerable attention over the last 20 years, both from an analysis and numerical simulation point of view, see, e.g., [1,3,4,11,12,18,25]. It is the equilibrium Freundlich isotherm, however, which makes the problem most challenging due to the degeneracy/singularity.

In the multi-species case (transport of several contaminants), $\mathbf{z} = (z_1, \dots, z_N)$, is a vector-valued function but the evolution of the concentration of each component is still described by (1.1). The adsorption process is now competitive (different species competing for the same adsorption sites), thus leading to a coupled system of PDEs, and the competitive adsorption process is modeled by a multicomponent isotherm, see [1,20,23,24,28] for a review of competitive equilibrium adsorption modeling. The most common multicomponent adsorption isotherms $\mathbf{b}_a(\mathbf{z}) = (b_{a1}(\mathbf{z}), \dots, b_{aN}(\mathbf{z}))$ are

$$\begin{aligned} b_{ai}(\mathbf{z}) &= K_i \left(\sum_{j=1}^N \alpha_{ij} z_j \right)^{p_i-1} z_i && \text{Freundlich,} \\ b_{ai}(\mathbf{z}) &= K_i \frac{1}{1 + \sum_{j=1}^N \alpha_{ij} z_j} z_i && \text{Langmuir,} \end{aligned}$$

where K_i, α_{ij}, p_i are non-negative parameters. The α_{ij} are dimensionless competition coefficients describing the inhibition of species i to the adsorption of species j . By definition $\alpha_{ii} = 1$ and the values for α_{ij} vary normally between zero (no competition between species i and j) and ten, cf. [17].

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