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Well-posedness and dynamics of the stochastic fractional magneto-hydrodynamic equations[☆]

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The current paper is devoted to the well-posedness and dynamics of the stochastic 2D incompressible fractional Magneto-Hydrodynamic(MHD) equations driven by Gaussian multiplicative noise. The nonlocal fractional diffusion leads to a new difficulty in the convergence since higher order estimates cannot be obtained. The commutator estimates are introduced to overcome these difficulties. Using the stopping time technique and monotonicity arguments, the global existence and uniqueness of the weak solution are obtained in a fixed probability space. Finally, the existence of a random attractor for the random dynamical systems generated by the solution of stochastic MHD equation is presented.

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1. Introduction

Recently, the fractional partial differential equations appear more and more frequently in different research areas and engineering applications. They have been applied to model various phenomena in image analysis, risk management and statistical mechanics(see e.g. [2,3]). There have been extensive study and application of fractional differential equations including the fractional Schrödinger equation [10], fractional Landau–Lifshitz equation [14,21], fractional Landau–Lifshitz Gilbert equation [20], and fractional Landau–Lifshitz–Maxwell equation [13].

The dynamics of the velocity and the magnetic field in electrically conducting fluids and some basic physics conservation laws can be described by the MHD equations. More details of the related background can be referred to [4,7,17]. There have been extensive study of the MHD equations in the following

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form

$$\begin{cases}\partial_t u + u\cdot\nabla u = -\nabla p + \nu_1\partial_{x_1}^2 u + \nu_2\partial_{x_2}^2 u + b\cdot\nabla b,\\ \partial_t b + u\cdot\nabla b = \eta_1\partial_{x_1}^2 b + \eta_2\partial_{x_2}^2 b + b\cdot\nabla u,\\ \nabla\cdot u = 0, \qquad \nabla\cdot b = 0,\\ u(0,x) = u_0, \qquad b(0,x) = b_0,\end{cases} \tag{1.1}$$

where $(x_1, x_2) \in \mathbb{R}^2$, $t \geq 0$, $u = (u_1, u_2)$ and $b = (b_1, b_2)$ denote the velocity field and magnetic field respectively, p is a scalar pressure, $\nu_1, \nu_2 \geq 0$ is the kinematic viscosity, $\eta_1, \eta_2 \geq 0$ is the magnetic diffusion. M. Sermange and R. Temam in [23] and G. Duvaut and J.L. Lions in [12] shown the existence and uniqueness of the global solution corresponding to sufficiently smooth initial data for (1.1) for all parameters $\nu_1, \nu_2, \eta_1, \eta_2 > 0$, see Theorem 6 in [12]. Also, when some of the parameters are positive, the well-posedness of (1.1) was obtained in [6,18]. J. Wu in [25] and Y. Zhou in [26] studied the regularity of the solution for the generalized fractional MHD equations.

Recently, the authors in [5] obtained the global regularity of (1.1) with fractional operator on the case that $\nu_1 > 1, \nu_2 = 0$ and $\eta_1 > 1, \eta_2 = 0$ in the following form

$$\begin{cases}\partial_t u + u\cdot\nabla u + \varepsilon(-\triangle)^\alpha u = -\nabla p + \partial_{x_1}^2 u + b\cdot\nabla b,\\ \partial_t b + u\cdot\nabla b + \varepsilon(-\triangle)^\alpha b = \partial_{x_1}^2 b + b\cdot\nabla u,\\ \nabla\cdot u = 0, \qquad \nabla\cdot b = 0,\\ u(0,x) = u_0, \qquad b(0,x) = b_0,\end{cases} \tag{1.2}$$

with $\varepsilon > 0$ and $\alpha > 0$. For the two dimensional stochastic MHD equations

$$\begin{cases}dX = \left(\nu\Delta X - (X\cdot\nabla)X + S(B\cdot\nabla)B - \nabla\left(P + \frac{1}{2}S|B|^2\right)\right)dt + \sqrt{Q_1}dW_1(t),\\ dB = (\nu_1\Delta B - (X\cdot\nabla)B + (B\cdot\nabla)X)dt + \sqrt{Q_2}dW_2(t),\\ \nabla\cdot X = 0, \qquad \nabla\cdot B = 0, B\cdot n = 0, \quad \text{in } (0,+\infty)\times\mathbb{O},\\ X = 0, \qquad curl\ B = 0, \quad \text{on } (0,+\infty)\times\partial\mathbb{O},\\ X(0,\xi) = x_0(\xi), \qquad B(0,\xi) = b_0(\xi), \quad \text{on } \mathbb{O}.\end{cases} \tag{1.3}$$

Barbu and Da Prato in [1] showed the existence of the solution to the stochastic MHD equations (1.3), and proved the existence and uniqueness of an invariant measure by coupling methods.

There exists a natural relationship between the fractional Laplace operator $(-\Delta)^\alpha$ and some special stochastic process. Let $X(t, x_0, \omega)$ be the solution process of a stochastic partial differential equation. For observable $\phi(\cdot)$, we can define the Markov semigroup $\{P_t\}_{t\geq 0}$ by

$$P_t\phi(X_t) = \mathbb{E}\phi(X_t), \qquad P_{t+s} = P_t P_s.$$

The infinitesimal generator is the derivative of semigroup P_t at time $0 : A\phi(X) = \frac{d}{dt}P_t\phi(X)$, where the infinitesimal generator A carries information about stochastic process X_t. It is well known that the infinitesimal generator of Brownian motion is Laplacian operator Δ. For α stable Lévy motion L_t^α with $0 < \alpha \leq 2$, by Lévy-Khintchine theorem, its jump measure is $\nu_\alpha(dy) = C_\alpha\frac{dy}{|y|^{1+\alpha}}$, and the generator of α-stable Levy noise is just the fractional Laplacian operator $(-\Delta)^{\frac{\alpha}{2}}$. For example, the nonlocal operator such as pseudo-partial differential operator has the following form

$$\int_{R^d\backslash 0}[u(x+y) - u(x) - I_{\{|y|<1\}}yu'(x)]\nu_\alpha(dy) = -K_\alpha(-\Delta)^{\frac{\alpha}{2}}$$

where $\nu_\alpha(dy) = C_\alpha\frac{dy}{|y|^{d+\alpha}}$ is the jump measure for the α-stable Levy noise. Hence, $(-\Delta)^\alpha$ is nonlocal except when $\alpha = 0, 1, 2, \ldots$, it is a pseudo-differential operator whose symbol is $|\xi|^\alpha$, and can be realized through the Fourier transform

$$\widehat{(-\Delta)^\alpha f}(\xi) = |\xi|^{2\alpha}\hat{f}(\xi).$$

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