Nonlinear Analysis 133 (2016) 200–227

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

Flux approximation to the isentropic relativistic Euler equations☆

Hanchun Yang*, Yu Zhang

Department of Mathematics, Yunnan University, Kunming, 650091, PR China

HIGHLIGHTS

- The isentropic relativistic Euler equations under flux perturbations are studied.
- A family of delta-shock and U-shaped pseudo-vacuum solutions are constructed.
- The vanishing pressure and flux approximation limits are analyzed respectively.
- The flux approximations have their respective effects on the delta-shock and vacuum.

ARTICLE INFO

Article history:
Received 12 July 2015
Accepted 4 December 2015
Communicated by Enzo Mitidieri

Keywords:
Isentropic relativistic Euler equations
Pressureless relativistic Euler equations
Delta shock wave
Vacuum
Flux approximation
Lorentz transformation

ABSTRACT

The isentropic relativistic Euler equations for polytropic gas under flux perturbations are studied. The Riemann problem of the pressureless relativistic Euler equations with a flux approximation is firstly solved, and a family of delta-shock and U-shaped pseudo-vacuum state solutions are constructed. Then it is shown that, as the flux approximation vanishes, the limits of the family of delta-shock and U-shaped pseudo-vacuum solutions are exactly the delta-shock and vacuum state solutions to the pressureless relativistic Euler equations, respectively. Secondly, we study the Riemann problem of the isentropic relativistic Euler equations with a double parameter flux approximation including pressure term. We further prove that, as the pressure and two-parameter flux perturbation vanish, respectively, any two-shock Riemann solution tends to a delta-shock solution to the pressureless relativistic Euler equations, and the intermediate density between the two shocks tends to a weighted δ-measure which forms a delta shock wave; any two-rarefaction Riemann solution tends to a two-contact-discontinuity solution to the pressureless relativistic Euler equations, and the nonvacuum intermediate state in between tends to a vacuum state.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

Since Taub's fundamental work [29] in 1948, the relativistic fluid dynamics has been an attractive and challenging topic of the physicists and mathematicians due to its importance and extreme complexity. In

☆ Supported by the NSF of China (11361073).

* Corresponding author. Tel.: +86 871 65033702; fax: +86 871 65033700.
E-mail address: hyang@ynu.edu.cn (H. Yang).

http://dx.doi.org/10.1016/j.na.2015.12.002
0362-546X/© 2015 Elsevier Ltd. All rights reserved.

this paper, we are concerned with the Euler system of conservation laws of baryon numbers and momentum for a perfect isentropic fluid in special relativity [1,21,22,29,30]

$$\begin{cases} \left(\dfrac{n}{\sqrt{1-v^2/c^2}}\right)_t + \left(\dfrac{nv}{\sqrt{1-v^2/c^2}}\right)_x = 0, \\ \left(\dfrac{(\rho+p/c^2)v}{1-v^2/c^2}\right)_t + \left(\dfrac{(\rho+p/c^2)v^2}{1-v^2/c^2}+p\right)_x = 0, \end{cases} \tag{1.1}$$

which, taking the Newton limit $v/c \to 0$, turns to the classical Euler system for compressible isentropic fluids (see [5–7,12,23,26]) etc., where ρ and v represent the proper energy density and particle speed, respectively, $p = p(\rho)$ is the pressure, n the proper number density of baryons and c the speed of light. ρ and v are in the physically relevant region $\{(\rho, v)|\ \rho \geq 0,\ |v| < c\}$. According to the first law of thermodynamics, n is determined by

$$\theta dS = \frac{1}{n}d\rho - \frac{\rho + p/c^2}{n^2}dn, \tag{1.2}$$

where θ is the temperature and S the entropy per baryon. Thus, for isentropic fluids, it follows from (1.2) that

$$\frac{dn}{n} = \frac{d\rho}{\rho + p/c^2}.$$

Integrating the above expression, we have

$$n = n(\rho) = n_0 \exp\left(\int_1^\rho \frac{ds}{s + p(s)/c^2}\right), \quad (n_0 \text{ is a constant}). \tag{1.3}$$

For polytropic gas, the state equation has the form

$$p(\rho) = \kappa^2 \rho^\gamma, \quad \gamma > 1, \tag{1.4}$$

where κ is a positive constant satisfying $\kappa < c$. Specifically, $p = \kappa^2\rho$ models an isothermal gas, which corresponds to the extremely relativistic gas, when the temperature is very high and particles move near the speed of light. This case will be studied in the future.

On the system (1.1) there are few considerable studies and progresses. For instance, taking $c = 1$, Pant [24] solved the Riemann problem and Cauchy problem for the case $\gamma = 1$. Chen and Li [8] established the uniqueness of Riemann solutions in the class of entropy solutions with arbitrarily large oscillation. Li and Geng [18] discussed the convergence of the entropy solutions as $c \to \infty$ for $\gamma = 1$, based on the geometric properties of nonlinear wave curves and the Glimm's method. Ding and Li [13] studied the global existence and non-relativistic global limits of entropy solutions to the one dimensional piston problem. Cheng and Yang [11] solved the Riemann problem of (1.1) with a Chaplygin gas pressure law.

Letting the pressure $p = 0$ in the isentropic relativistic Euler equations (1.1) yields the pressureless relativistic Euler equations

$$\begin{cases} \left(\dfrac{\rho}{\sqrt{1-v^2/c^2}}\right)_t + \left(\dfrac{\rho v}{\sqrt{1-v^2/c^2}}\right)_x = 0, \\ \left(\dfrac{\rho v}{1-v^2/c^2}\right)_t + \left(\dfrac{\rho v^2}{1-v^2/c^2}\right)_x = 0, \end{cases} \tag{1.5}$$

which, in the Newton limit, reduces to the zero-pressure gas dynamics (see [2–4,41,14,20,19,25]) etc. The system (1.5) is fully linearly degenerate, so the elementary waves involve only contact discontinuities. What interesting is that both delta shock waves and vacuum states do occur in solutions. As for the delta shock wave, there have been rich results and we refer to [15,16,25,27,28,31,32,35,37,38] and the references cited therein.

Download English Version:

https://daneshyari.com/en/article/839337

Download Persian Version:

https://daneshyari.com/article/839337

Daneshyari.com