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Fractal dimension of random invariant sets for nonautonomous random dynamical systems and random attractor for stochastic damped wave equation[☆]

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ABSTRACT

In this paper, we first establish a criteria for estimating the upper bound of fractal dimension of a random compact invariant set of a non-autonomous random dynamical system on a separable Banach space and give some special conditions which can be easily verified to some stochastic partial differential equations for bounding the fractal dimension of random attractors. Then we apply these conditions to obtain an upper bound of fractal dimension of random attractor for stochastic damped wave equation with additive white noise.

1. Introduction

It is well known that the existence of attractor and the estimate of its dimension are two main problems in studying the asymptotic behavior of infinite-dimensional dynamical systems. Finite dimensionality is an important property of compact invariant sets for dynamical systems. The Hausdorff and fractal dimensions of the global attractor, pullback attractor (or kernel sections) for deterministic autonomous and non-autonomous dynamical systems were studied widely, see [2,9,18,19,23,25,35]. Recently, there have been many publications concerning the random attractors for random dynamical systems, of those, most works are to study the existence of random attractors, see [3,4,6,10,12,13,11,30,29,28,27] and the references wherein. For

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example, Crauel, Flandoli, Debussche and Chueshov [10,12,11] established a theory for the existence of random attractors for autonomous stochastic systems. For non-autonomous stochastic evolution equations with the time-dependent external term and white noise, Wang established a useful theory about the existence and upper semi-continuity of random attractors by introducing two parametric spaces and given some applications to non-autonomous stochastic reaction–diffusion equations and wave equations, see [30,29,28,27]. As we know, up to now, there are several effective approaches to estimate the upper bound of Hausdorff and fractal dimension of random attractors, see [13–15,20,21,31]. Crauel and Flandoli [13] developed a technique for bounding the Hausdorff dimension of random attractors requiring the differentiability of random dynamical system and the boundedness of noise; Debussche [15] provided a method to estimate the Hausdorff dimension of a random invariant set of a random map by using a random squeezing property and Lyapunov exponents similar to [13]; and Langa [21] generalized the method of [15] to the fractal dimension. Wang and Tang [31] gave a method to bound the fractal dimension of random attractor similar to [21], but requiring some "strong" conditions that the Lipschitz constant of system and the "contraction" coefficient of the infinite-dimensional part of system are deterministic constants independent of the sample points which are only satisfied by some linear systems and some special nonlinear systems with uniform bounded derivative of the nonlinearity. The difficulty within a nonlinear random system is from the fact that a random attractor is not uniformly bounded with respect to the random parameter which resulting in the unboundedness of the nonlinear term and its derivatives. Debussche [14] overcame this difficulty and provided some conditions (similar to the conditions of Theorem 2.2) to estimate the Hausdorff dimension of a random attractor by making use of boundedness of the attractor in the mean.

The finite fractal dimension of attractor plays a very important role in the finite-dimensional reduction theory of infinite dimensional dynamical systems based on the following fact: if A is a compact set in a metric space and the fractal dimension $\dim_f(A)$ of A is less than $m/2$ for some $m \in \mathbb{N}$, then there exists an injective Lipschitz mapping ϕ: $A \to \mathbb{R}^m$ such that its inverse is Hölder continuous. This means that A can be placed in the graph of Hölder continuous mapping which maps a compact subset of $\mathbb{R}^m$ onto A. But the finite Hausdorff dimension does not have such a property, that is, no finite parametrization is available for a set if only knowing the finiteness of its Hausdorff dimension (see [21,24]).

Motivated by the ideas of [35,29,14,21,31], in this article, we first establish a criteria for bounding the fractal dimension of a random invariant set under a non-autonomous random dynamical system and give some special conditions which can be easily verified to some stochastic partial differential equations originated from Debussche [14]. It is worth mentioning that the conditions in our criteria do not require the differentiability of random dynamical system and just need some boundedness of the random invariant set in the mean. And then as an example, we consider the following non-autonomous stochastic damped wave equation with additive white noise

$$\begin{cases} du_t + \alpha du + (-\Delta u + f(u,x))dt = g(x,t)dt + h(x)dW(t) \quad \text{in } U \times (\tau, +\infty),\ \tau \in \mathbb{R}, \\ u(x,t)|_{x\in\partial U} = 0, \quad t \geq \tau, \\ u(x,\tau) = u_\tau(x), \qquad u_t(x,\tau) = u_{1\tau}(x), \quad x \in U, \end{cases} \tag{1.1}$$

where $u = u(x,t)$ is a real-valued function on $U \times [\tau, +\infty)$, $\tau \in \mathbb{R}$, U is an open bounded set of $\mathbb{R}^n$ $(n \leq 3)$ with a smooth boundary ∂U, $g(x,\cdot) \in C_b(\mathbb{R},\ H_0^1(U))$, $h(\cdot) \in H_0^1(U) \cap H^2(U)$, $\alpha > 0$, $f(\cdot,\cdot)$ satisfies some dissipative conditions, $W(t)$ is a one-dimensional two-sided Wiener process.

For the random system (1.1) with g being independent of t, Crauel, Debussche, Flandoli, Lv and Yang et al. proved the existence of a random attractor in [11,22,32]. X. Fan [16,17] and Zhou et al. [34] obtained an upper bound of the Hausdorff dimension of the random attractor by using the method of [15]. Debussche [14] proved the boundedness of Hausdorff dimension of the random attractor for (1.1). Here we apply our criteria to obtain an upper bound of fractal dimension of random attractor for (1.1), which implies that the random attractor of (1.1) can be embedded in a finite dimensional Euclidean space.