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Critical growth fractional elliptic systems with exponential nonlinearity

J. Giacomoni[a,*], Pawan Kumar Mishra[b], K. Sreenadh[b]

[a] LMAP (UMR CNRS 5142) Bat. IPRA, Avenue de l'Université, F-64013 Pau, France
[b] Department of Mathematics, Indian Institute of Technology Delhi, Hauz Khaz, New Delhi-110016, India

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ABSTRACT

We study the existence of positive solutions for the system of fractional elliptic equations of the type,

$$\begin{aligned}(-\Delta)^{\frac{1}{2}}u &= \frac{p}{p+q}\lambda f(x) \mid u\mid^{p-2} u \mid v\mid^{q} + h_1(u,v)e^{u^2+v^2}, \quad \text{in } (-1,1),\\ (-\Delta)^{\frac{1}{2}}v &= \frac{q}{p+q}\lambda f(x) \mid u\mid^{p} \mid v\mid^{q-2} v + h_2(u,v)e^{u^2+v^2}, \quad \text{in } (-1,1),\\ u,v &> 0 \quad \text{in } (-1,1),\\ u &= v = 0 \quad \text{in } \mathbb{R}\setminus(-1,1)\end{aligned}$$

where $1 < p+q < 2$, $h_1(u,v) = (\alpha+2u^2) \mid u\mid^{\alpha-2} u \mid v\mid^{\beta}$, $h_2(u,v) = (\beta+2v^2) \mid u\mid^{\alpha} \mid v\mid^{\beta-2} v$ and $\alpha+\beta > 2$. Here $(-\Delta)^{\frac{1}{2}}$ is the fractional Laplacian operator. We show the existence of multiple solutions for suitable range of λ by analyzing the fibering maps and the corresponding Nehari manifold. We also study the existence of positive solutions for a superlinear system with critical growth exponential nonlinearity.

1. Introduction

We study the following system for existence and multiplicity of solutions

$$(P_\lambda)\quad \begin{cases}(-\Delta)^{\frac{1}{2}}u = \dfrac{p}{p+q}\lambda f(x)|u|^{p-2}u|v|^{q} + h_1(u,v)e^{u^2+v^2}, \quad \text{in } (-1,1),\\ (-\Delta)^{\frac{1}{2}}v = \dfrac{q}{p+q}\lambda f(x)|v|^{q-2}v|u|^{p} + h_2(u,v)e^{u^2+v^2}, \quad \text{in } (-1,1),\\ u,v > 0 \quad \text{in } (-1,1),\\ u = v = 0 \quad \text{in } \mathbb{R}\setminus(-1,1)\end{cases}$$

* Corresponding author.
E-mail addresses: jacques.giacomoni@univ-pau.fr (J. Giacomoni), pawanmishra31284@gmail.com (P.K. Mishra), sreenadh@gmail.com (K. Sreenadh).

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where $1 < p+q < 2$, $h_1(u,v) = (\alpha+2u^2)|u|^{\alpha-2}u|v|^\beta$ and $h_2(u,v) = (\beta+2v^2)|u|^\alpha|v|^{\beta-2}v$, $\alpha+\beta > 2$, $\lambda > 0$ and $f \in L^r(-1,1)$, for suitable choice of $r > 1$, is sign changing. Here $(-\Delta)^{\frac{1}{2}}$ is the $\frac{1}{2}$-Laplacian operator defined as

$$(-\Delta)^{\frac{1}{2}}u = \int_{\mathbb{R}} \frac{(u(x+y)+u(x-y)-2u(x))}{|y|^2}dy \quad \text{for all } x \in \mathbb{R}.$$

The fractional Laplacian operator has been a classical topic in Fourier analysis and nonlinear partial differential equations. Fractional operators are involved in financial mathematics, where Levy processes with jumps appear in modeling the asset prices (see [5]). Recently the semilinear equations involving the fractional Laplacian have attracted many researchers. The critical exponent problems for fractional Laplacian have been studied in [27,29]. Among the works dealing with fractional elliptic equations with critical exponents we cite also [33,8,23,24] and references there-in, with no attempt to provide a complete list.

In the local setting, the semilinear elliptic systems involving Laplace operator with exponential nonlinearity have been investigated in [13,22]. The case of polynomial nonlinearities involving linear and quasilinear operators has been studied in [3,4,12,14,16,28,30]. Furthermore, these results for sign changing nonlinearities with polynomial type subcritical and critical growth have been obtained in [6,7,11,26,31] using Nehari manifold and fibering map analysis. These problems for exponential growth nonlinearities is studied in [13]:

$$\begin{cases} -\Delta u = g(v), \qquad -\Delta v = f(u), \quad \text{in } \Omega \\ \quad u = v = 0 \quad \text{on } \partial\Omega \end{cases}$$

where functions f and g have critical growth in the sense of Trudinger–Moser inequality and have shown the existence of a nontrivial weak solutions in both sub-critical as well as critical growth case. In [22], authors have considered the elliptic system with exponential growth perturbed by a concave growth term and established the global multiplicity results with respect to the parameter.

Recently semilinear equations involving fractional Laplacian with exponential nonlinearities have been studied by many authors. Among them we cite [19,21,32,17] and the references therein. The system of equations with fractional Laplacian operator with polynomial subcritical and critical Sobolev exponent have been studied in [10,18]. Our aim in this article is to generalize the result in [17] for fractional elliptic systems. In [18], authors considered the problem

$$(-\Delta)^s u = \lambda|u|^{q-2}u + \frac{2\alpha}{\alpha+\beta}|u|^{\alpha-2}u|v|^\beta, \qquad (-\Delta)^s = \lambda|v|^{q-2}v + \frac{2\alpha}{\alpha+\beta}|u|^\alpha|v|^{\beta-2}v \quad \text{in } \Omega,$$
$$u = v = 0 \quad \text{on } \partial\Omega$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $\lambda, > 0, 1 < q < 2$ and $\alpha > 1, \beta > 1$ satisfy $\alpha+\beta = 2N/(N-2s), s \in (0,1)$ and $N > 2s$. They studied the associated Nehari manifold using the fibering maps and show the existence of non-negative solutions arising out of structure of manifold. Our results in Section 3 extends these results to the exponential case.

The variational functional J_λ associated to the problem (P_λ) is given as

$$J_\lambda(u,v) = \frac{1}{2}\int_{-1}^{1}\left(|(-\Delta)^{\frac{1}{4}}u|^2 + |(-\Delta)^{\frac{1}{4}}v|^2\right)dx - \frac{\lambda}{p+q}\int_{-1}^{1} f(x)|u|^p|v|^q dx - \int_{-1}^{1} G(u,v)dx.$$

Definition 1.1. $(u,v) \in H_0^s(-1,1) \times H_0^s(-1,1)$ is called weak solution of (P_λ) if

$$\int_{-1}^{1}\left((-\Delta)^{\frac{1}{4}}u(-\Delta)^{\frac{1}{4}}\phi + (-\Delta)^{\frac{1}{4}}v(-\Delta)^{\frac{1}{4}}\psi\right)dx = \lambda\int_{-1}^{1}\left(|u|^{p-2}|v|^q u\phi + |v|^{q-2}|u|^p v\psi\right)dx + \int_{-1}^{1}\left(h_1(u,v)\phi + h_2(u,v)\psi\right)e^{u^2+v^2}dx$$

for all $(\phi,\psi) \in H_0^s(-1,1) \times H_0^s(-1,1)$.