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On forced oscillations in groups of interacting nonlinear systems

Ivan Polekhin

Steklov Mathematical Institute, 119991 Moscow, Russia

HIGHLIGHTS

- We study periodically forced motion in groups of interacting systems.
- We present sufficient conditions for the existence of a periodic solution.
- The result is obtained as an application of a topological approach.

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ABSTRACT

Consider a periodically forced nonlinear system which can be presented as a collection of smaller subsystems with pairwise interactions between them. Each subsystem is assumed to be a massive point moving with friction on a compact surface, possibly with a boundary, in an external periodic field. We present sufficient conditions for the existence of a periodic solution for the whole system. The result is illustrated by a series of examples including a chain of strongly coupled pendulums in a periodic field.

1. Brief introduction

The phenomenon of forced oscillations has been studied since at least 1922, when G. Hamel proved [8] that the equation describing the motion of a periodically forced pendulum has at least one periodic solution. Various results, which generalize and develop [8], have appeared in the literature since this paper; see, for example, [2,5,7,6,13]. Besides this, forced oscillations in a system of coupled planar pendulums and its generalizations have been studied in [11,12] under the assumption of certain symmetry properties for the forcing terms. The existence and multiplicity of periodic solutions for coupled systems is also discussed in [3]. In [1], an important case, which includes various forced systems composed by a finite number of interacting particles, is considered and conditions for the existence of an asymptotically stable forced oscillation are obtained.

We present a theorem which is also related to the study of the existence of periodic solutions in non-autonomous systems. Our result can be useful, among other things, in studying forced oscillations in a

E-mail address: ivanpolekhin@gmail.com.

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nonlinear lattice, which is a cornerstone model in nonlinear physics and is widely used for analytical and computational purposes [9,16,4,10]. The result continues a previously reported work [14].

In the paper, the following systems are considered. Let us have several compact smooth manifolds, possibly with boundaries, with non-zero Euler–Poincaré characteristics. Suppose that for each manifold there is a massive point moving on it with viscous friction. All points are in an external periodic field and may interact with each other. It is allowed that the interactions may be of different types and also may be arbitrarily strong. We present sufficient conditions for the existence of a periodic solution for such a system.

We would like to note that when the considered manifolds are closed, it is not hard to prove that there exists a periodic solution in the system: it directly follows from the Lefschetz–Hopf theorem. Yet in applications it appears to be useful to consider manifolds with boundaries to prove the existence of a periodic solution and estimate it. One of the examples of such kind, which we consider further below, is widely used in nonlinear studies model of coupled oscillators on a line. Therefore, our result generalizes the approach based on an application of the Lefschetz–Hopf theorem to the important case of compact manifolds with boundaries.

Our result is based on a theorem by R. Srzednicki, K. Wójcik, and P. Zgliczyński [15] and provides an illustrative geometrical approach to study periodical oscillations. Since the result is formulated in a coordinate-free form, it also avoids the possible shortcomings of purely analytical approaches and can be relatively easy applied to systems with complex topology of the phase space.

The main theorem is illustrated by a series of examples from mechanics including a system of an arbitrary number of strongly coupled pendulums in a periodic external field.

2. Main result

2.1. Governing equations

The equations introduced in this subsection – which we are going to use further below in the paper – generalize the governing equations for a mechanical system consisting of massive points moving with friction-like interaction on compact surfaces. For the sake of simplicity, we assume that all manifolds and considered functions are smooth (i.e. C^∞).

Let M_i be a compact connected one- or two-dimensional manifold (possibly with a boundary), where $i = 1, \ldots, n$. For all $i = 1, \ldots, n$, there is a point moving in an external force field on the manifold M_i. We also assume that there is a friction-like force acting on the point.

In our further consideration, we will study the behavior of the system in vicinities of ∂M_i and it will be convenient to consider enlarged manifolds M_i^+. Let M_i^+ be a boundaryless connected manifold such that $M_i \subset M_i^+, \dim M_i = \dim M_i^+$. Also suppose that every manifold M_i^+ is equipped with a smooth Riemannian metric $\langle \cdot, \cdot \rangle_i$.

If the points do not interact, one can consider the following independent equations of motion

$$\nabla^i_{\dot{q}_i} \dot{q}_i = f_i(t, q_i, \dot{q}_i) + f_i^{friction}(t, q_i, \dot{q}_i), \quad i = 1, \ldots, n, \tag{1}$$

where ∇^i means the covariant differentiation with respect to the corresponding metric $\langle \cdot, \cdot \rangle_i$ for the ith point moving on M_i^+; $f_i \colon \mathbb{R}/T\mathbb{Z} \times TM_i^+ \to TM_i^+$ and $f_i(t, q_i, \dot{q}_i) \in T_{q_i} M_i^+$ for any $t, q_i, \dot{q}_i$; $f_i^{friction} \colon \mathbb{R}/T\mathbb{Z} \times TM_i^+ \to TM_i^+$ corresponds to a friction-like force.

As we said before, (1) includes the case of a mechanical system of massive points in an external field. Indeed, in this case $\langle \cdot, \cdot \rangle_i$ is the corresponding kinetic metric and, for given t, q_i and $\dot{q}_i, f_i(t, q_i, \dot{q}_i)$ and $f_i^{friction}(t, q_i, \dot{q}_i)$ are the dual vectors to the corresponding generalized forces. Note that from (1) it follows that if there is no forces acting on the ith point then (1) becomes $\nabla^i_{\dot{q}_i} \dot{q}_i = 0$, which is the equation of the geodesic motion.