Nonlinear Analysis 132 (2016) 1–24

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

Incompressible limit of a compressible micropolar fluid model with general initial data

Jingrui Su

Department of Mathematics, Nanjing University, Nanjing 210093, PR China

A R T I C L E   I N F O

Article history:
Received 15 July 2015
Accepted 21 October 2015
Communicated by Enzo Mitidieri

MSC:
76W05
35B40

Keywords:
Compressible micropolar fluids
Incompressible micropolar fluids
Low Mach number limit
Convergence rate

A B S T R A C T

In this paper we consider the incompressible limit of a compressible micropolar fluid model with general initial data in the whole space $\mathbb{R}^3$. It is proved that, as the Mach number goes to zero, the weak solutions of the compressible micropolar fluids converge to the strong solution of the ideal incompressible micropolar fluids as long as the latter exists. Moreover, the rate of convergence is also obtained.

1. Introduction

Micropolar fluids are fluids with microstructure characterized by an asymmetric stress tensor. Micropolar fluid models are widely used to deal with the fluids consisting of randomly oriented particles suspended in a viscous medium such as animal blood's motion, liquids crystal and dilute aqueous polymer solutions [25].

The micropolar fluid model was firstly proposed by Eringen [15] in 1966. Subsequently, various micropolar fluid models were derived to describe certain physical processes or phenomenon. Generally speaking, there are two kinds of micropolar fluid models: incompressible micropolar fluid models and compressible micropolar fluid models. For the incompressible micropolar fluid models, its mathematical theory is rather richer. There are a lot of works on the existence of weak solution, local-in-time strong solution for large initial data, global existence and large time behavior of strong solution for small initial data, and regularity criteria of strong solutions, see [4,6,7,12,26,33,25,30–32] and the references cited therein.

On the other hand, the compressible micropolar fluid models are more complex than the incompressible ones and only a few results are available. Easwaran and Majumdar [14] obtained a uniqueness result for compressible micropolar model in the whole space $\mathbb{R}^1$. Mujaković [28,29] obtained the local and global

E-mail address: sjr-sujingrui@163.com.

http://dx.doi.org/10.1016/j.na.2015.10.020

existence as well as stabilization and regularity of solutions for one dimensional compressible micropolar model with inhomogeneous boundary conditions. Drazić and Mujaković [13] proved a local existence result for a 3D compressible micropolar fluid with spherical symmetry. Chen et al. [3,5] established some blow-up criteria of strong solutions to the compressible viscous micropolar fluids. Amirat and Hamdache [1] proposed the global-in-time existence of weak solutions for the compressible magneto-micropolar fluids, which includes compressible micropolar fluids as a special case, with finite energy.

From the physical point of view, incompressible models can be derived from the corresponding compressible ones when the Mach number goes to zero and the density becomes almost a constant. To the best of our knowledge, there is no result on the zero Mach number limit of the compressible micropolar fluids models. The goal of this paper is to study this topic.

We focus our attention on the following compressible viscous micropolar fluids (see [1,13]):

$$\partial_t\tilde\rho + \operatorname{div}(\tilde\rho\tilde{\mathbf{u}}) = 0, \tag{1.1}$$

$$\partial_t(\tilde\rho\tilde{\mathbf{u}}) + \operatorname{div}(\tilde\rho\tilde{\mathbf{u}}\otimes\tilde{\mathbf{u}}) + \nabla\tilde P(\tilde\rho) = (\tilde\mu+\tilde\zeta)\Delta\tilde{\mathbf{u}} + (\tilde\mu+\tilde\lambda-\tilde\zeta)\nabla(\operatorname{div}\tilde{\mathbf{u}}) + 2\tilde\zeta\operatorname{curl}\tilde\omega, \tag{1.2}$$

$$\partial_t(\tilde\rho\tilde\omega) + \operatorname{div}(\tilde\rho\tilde{\mathbf{u}}\otimes\tilde\omega) + 4\tilde\zeta\tilde\omega = \tilde{\mu'}\Delta\tilde\omega + (\tilde{\mu'}+\tilde{\lambda'})\nabla(\operatorname{div}\tilde\omega) + 2\tilde\zeta\operatorname{curl}\tilde{\mathbf{u}}. \tag{1.3}$$

Here $\tilde\rho$ denotes the density, $\tilde{\mathbf{u}}\in\mathbb{R}^3$ the velocity, $\tilde\omega\in\mathbb{R}^3$ the angular velocity of rotation of particles of the fluid (also called microrotation field), and $\tilde P(\tilde\rho)$ the pressure respectively. The parameter $\tilde\mu$ stands for shear viscosity and $\tilde\lambda$ the buck viscosity. The parameters $\tilde{\mu'},\tilde{\lambda'}$ and $\tilde\zeta$ are the coefficients of micro-viscosity. The viscosity coefficients of the fluids $\tilde\mu$, $\tilde\lambda$, $\tilde\zeta$, $\tilde{\mu'}$ and $\tilde{\lambda'}$ satisfy

$$\tilde\mu>0,\qquad \tilde{\mu'}>0,\qquad \tilde\zeta>0,\qquad 2\tilde\mu+3\tilde\lambda-4\tilde\zeta\ge 0,\qquad 2\tilde{\mu'}+3\tilde{\lambda'}\ge 0. \tag{1.4}$$

For sake of simplicity, we consider the pressure–density function $\tilde P(\tilde\rho)$ as

$$\tilde P(\tilde\rho) = a\tilde\rho^\gamma,\quad a>0,\ \gamma>\frac{3}{2}. \tag{1.5}$$

To study the low Mach number limit to the system (1.1)–(1.3), we introduce the following scalings for the unknowns

$$\tilde\rho(x,t) = \rho^\epsilon(x,\epsilon t),\qquad \tilde{\mathbf{u}}(x,t) = \epsilon\mathbf{u}^\epsilon(x,\epsilon t),\qquad \tilde\omega(x,t) = \epsilon\omega^\epsilon(x,\epsilon t) \tag{1.6}$$

and for the viscosity coefficients

$$\tilde\mu = \epsilon\mu^\epsilon,\qquad \tilde\lambda = \epsilon\lambda^\epsilon,\qquad \tilde\zeta = \epsilon\zeta^\epsilon,\qquad \tilde{\mu'} = \epsilon(\mu')^\epsilon,\qquad \tilde{\lambda'} = \epsilon(\lambda')^\epsilon, \tag{1.7}$$

where $\epsilon\in(0,1)$ is a small parameter (scaled Mach number). Thanks to (1.4), we know that the normalized coefficients $\mu^\epsilon,\lambda^\epsilon,\zeta^\epsilon,(\mu')^\epsilon$ and $(\lambda')^\epsilon$ satisfy

$$\mu^\epsilon>0,\qquad (\mu')^\epsilon>0,\qquad \zeta^\epsilon>0,\qquad 2\mu^\epsilon+3\lambda^\epsilon-4\zeta^\epsilon\ge 0,\qquad 2(\mu')^\epsilon+3(\lambda')^\epsilon\ge 0. \tag{1.8}$$

In view of the above-mentioned scalings and the pressure function (1.5), the system (1.1)–(1.3) can be rewritten as

$$\partial_t\rho^\epsilon + \operatorname{div}(\rho^\epsilon\mathbf{u}^\epsilon) = 0, \tag{1.9}$$

$$\partial_t(\rho^\epsilon\mathbf{u}^\epsilon) + \operatorname{div}(\rho^\epsilon\mathbf{u}^\epsilon\otimes\mathbf{u}^\epsilon) + \frac{a\nabla(\rho^\epsilon)^\gamma}{\epsilon^2} = (\mu^\epsilon+\zeta^\epsilon)\Delta\mathbf{u}^\epsilon + (\mu^\epsilon+\lambda^\epsilon-\zeta^\epsilon)\nabla(\operatorname{div}\mathbf{u}^\epsilon) + 2\zeta^\epsilon\operatorname{curl}\omega^\epsilon, \tag{1.10}$$

$$\partial_t(\rho^\epsilon\omega^\epsilon) + \operatorname{div}(\rho^\epsilon\mathbf{u}^\epsilon\otimes\omega^\epsilon) + 4\zeta^\epsilon\omega^\epsilon = (\mu')^\epsilon\Delta\omega^\epsilon + ((\mu')^\epsilon+(\lambda')^\epsilon)\nabla(\operatorname{div}\omega^\epsilon) + 2\zeta^\epsilon\operatorname{curl}\mathbf{u}^\epsilon. \tag{1.11}$$

Formally, it follows from the momentum equation (1.10) that ρ^ϵ converges to some function $\bar\rho(t)\ge 0$ when $\epsilon\to 0$. Furthermore, if we suppose that the initial datum ρ_0^ϵ is of order $1+O(\epsilon)$ (it is possible to arrive it from the initial energy (2.5) below), then we can expect that $\bar\rho=1$. If the limits $\mathbf{u}^\epsilon\to\mathbf{u}$ and $\omega^\epsilon\to\omega$ exist, then we can obtain $\operatorname{div}\mathbf{u}=0$ from the continuity equation (1.9). Assume further that

$$\mu^\epsilon\to 0,\qquad \lambda^\epsilon\to 0,\qquad \zeta^\epsilon\to 0,\qquad (\mu')^\epsilon\to 0,\qquad (\lambda')^\epsilon\to 0\quad \text{as } \epsilon\to 0.$$