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Stability of traveling waves in a population dynamics model with spatio-temporal delay

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ABSTRACT

In this paper, we are concerned with the stability of all traveling waves for a population dynamics model with spatio-temporal delay. By the weighted-energy method combining comparison principle, the global exponential stability of all traveling waves for the model is established, even including the slower waves whose wave speed is close to the critical speed.

1. Introduction

As is well known, reaction–diffusion equations are often used to model and describe the practical problems we meet in ecology, biology, population dynamics, chemistry, and so on. For example, based on the experimental data of Nicholson for the population of the Australian sheep-blowfly, Gurney et al. [4] originally established the following population dynamics model without diffusion, which is the so-called Nicholson's blowflies model

$$\frac{du(t)}{dt} = -\delta u(t) + pu(t-\tau)e^{-au(t-\tau)},$$

where $\delta > 0$ is the death rate of the mature population, $\tau > 0$ is the maturation delay, the time required for a newborn to become matured, and $a > 0$ is a constant. $p > 0$ is the impact of the death on the immature population and $u(t-\tau)e^{-au(t-\tau)}$ is Nicholson's birth function. Considering spatial variability of blowflies,

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many problems are investigated about the diffusive Nicholson's blowflies model

$$\frac{\partial u(t,x)}{\partial t} = \frac{\partial^2 u(t,x)}{\partial x^2} - \delta u(t,x) + pu(t-\tau,x)e^{-au(t-\tau,x)},$$

see [21–23]. However, more natural attention is paid to the modeling of the time delays to incorporate associated non-local spatial terms which account for the drift of blowflies to their present position from their possible positions at previous time, i.e., the following Nicholson's blowflies model with spatio-temporal delay

$$\frac{\partial u(t,x)}{\partial t} = \frac{\partial^2 u(t,x)}{\partial x^2} - \delta u(t,x) + p(g*u)(t,x)e^{-a(g*u)(t,x)}. \tag{1.1}$$

The convolution $g*u$ is denoted by

$$(g*u)(t,x) = \int_{-\infty}^{t}\int_{-\infty}^{+\infty} g(t-s,x-y)u(s,y)dyds,$$

and $g(t,x)$ denote different kinds of spatio-temporal delay kernels which are usually seen in some literature [9,10,24]. For example, when $(g*u)(t,x) = \int_{-\infty}^{t} g(t-s)u(s,x)ds$ and $g(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}$ or $g(t) = \frac{t}{\tau^2}e^{-\frac{t}{\tau}}$, Gourley [3] proved the existence of traveling wave fronts for Eq. (1.1) with those kernel functions using linear chain trick and geometric singular perturbation theory after there is a way to recast (1.1) into a non-delay finite dimensional ordinary differential system. Unfortunately, sometimes there is no way to recast (1.1) with special kernel functions into a non-delay finite dimensional ordinary differential system, such as

$$g(t,x) = \delta(t-\tau)\frac{1}{\sqrt{4\pi\rho}}e^{-\frac{x^2}{4\rho}}, \quad \tau > 0,\ \rho > 0. \tag{1.2}$$

Thus the methods of linear chain trick and geometric singular perturbation theory have no effect. Recently, Lin [10] employs the monotone iteration technique as well as the upper and lower solution method developed by Wang et al. [24] to obtain the existence of traveling wave solutions for (1.1) with the kernel functions (1.2).

In this paper, we continue to study (1.1) with the kernel function (1.2) and consider the Cauchy problem to (1.1) under initial conditions

$$u(s,x) = u_0(s,x), \quad s \in [-\tau, 0],\ x \in \mathbb{R}. \tag{1.3}$$

Namely, we provide a stability analysis of traveling wave solutions to (1.1). For reaction–diffusion equations with delay, to our knowledge, there are few results on the stability of traveling waves, see [8,12–17,19,20]. Schaaf [19] first established the local stability of traveling wave solutions by the spectrum analysis method for Fisher–Kpp nonlinearity and the equilibrium $u_- = 0$ is a stable node. Later, Smith and Zhao [20] obtained a globally exponential stability result for bistable nonlinearity by the "squeezing technique" in [1]. But the above methods cannot be applied to the nonlinearity in (1.1), because here $u_- = 0$ is an unstable node. Fortunately, this problem can be solved by introducing a proper weighted function [18] and adopting the weighted energy method developed by Mei [8,12–17]. Eq. (1.1) with the kernel function

$$g(t,x) = \delta(x)\delta(t-\tau) \tag{1.4}$$

has been extensively studied recently, see [12–17] and the references therein. By using the weighted energy method, Mei et al. [17] proved that the wavefronts of (1.1) with (1.4) are stable for the large wave speed (i.e., $c > c_*, c_*$ is the critical wave speed) and small initial perturbation. But for the small wave speed (i.e., c is close to c_*) and large initial perturbation (i.e., the initial perturbation around the wavefront decays to zero exponentially in space as $x \to -\infty$, but it can be allowed arbitrary large in other locations), the results (1.1) with (1.4) in [12–14] develop and improve the previous results in [16,17] by the help of new approach developed in [12] which is a combination of comparison principle and the technical weighted-energy method. Especially in Mei's [15], for nonlocal Nicholson's nonlinearity different from ours in this paper, Mei established the stability for all waves even including the slower waves whose wave speed is close to the critical