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Global strong solution for the density dependent incompressible viscoelastic fluids in the critical L^p framework☆

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ABSTRACT

In this paper, we consider the Cauchy problem of the density dependent incompressible viscoelastic fluids in the whole space $\mathbb{R}^d (d \geq 2)$ with the initial data close to a stable equilibrium. This work is dedicated to deriving the existence result of the global strong solution in the critical space with respect to the natural scaling of the equations in the L^p framework.

1. Introduction

In this paper, we want to investigate the Cauchy problem for the incompressible viscoelastic fluids with variable density, which is described by the following system:

$$\begin{cases} \partial_t \rho + u \cdot \nabla \rho = 0, \quad x \in \mathbb{R}^d, \ t > 0 \\ \partial_t(\rho u) + \nabla \cdot (\rho u \otimes u) - \mu \Delta u + \nabla \Pi = \nabla \cdot (\det(F)^{-1} F F^T), \\ \partial_t F + u \cdot \nabla F = \nabla u \cdot F, \\ \nabla \cdot u = 0, \qquad (\rho, u, F)|_{t=0} = (\rho_0, u_0, F_0), \end{cases} \tag{1.1}$$

where ρ and u represent the density and velocity of the fluid respectively, Π is the pressure and F is the deformation tensor introduced below. It is well known that elastic solids and viscous fluids are two extremes of material behavior. Viscoelastic fluids show intermediate behavior with some remarkable phenomena due to their elastic nature. They exhibit a combination of both fluid and solid characteristics and have received a great deal of interest. It can also be regarded as the consistence condition of the flow trajectories obtained from the velocity field u and also of those obtained from the deformation tensor F. Classically the motion of a fluid is described by a time-dependent family of orientations preserving diffeomorphism $X(t, x)$. Then

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deformation tensor F is defined as

$$F(t,x) = \frac{\partial X(t,x)}{\partial x}.$$

Applying the chain rule, we see that $F(t,x)$ satisfies the following transport equation (see [20]):

$$\partial_t F + u \cdot \nabla F = \nabla u \cdot F.$$

Throughout this paper, we will use the notations of

$$(\nabla v)_{i,j} = \frac{\partial v_i}{\partial x_j}, \qquad (\nabla v F)_{i,j} = (\nabla v)_{i,k} F_{k,j}, \qquad (\nabla \cdot F)_i = \partial_j F_{i,j},$$

and the summation over repeated indices will always be understood. We also assume that $a_0 = \frac{1}{\rho_0} - 1, E_0 = F_0 - I$ and u_0 satisfy the following constraints:

$$\det(E_0 + I) = 1, \qquad \nabla \cdot (E_0^T) = 0, \qquad \nabla \cdot u_0 = 0 \tag{1.2}$$

and

$$\partial_m E_{0ij} - \partial_j E_{0im} = E_{0lj} \partial_l E_{0im} - E_{0lm} \partial_l E_{0ij}. \tag{1.3}$$

Using these constraints we obtain that

$$\begin{cases} \det(E + I) = 1, \qquad \nabla \cdot (E^T) = 0, \\ \partial_m E_{ij} - \partial_j E_{im} = E_{lj} \partial_l E_{im} - E_{lm} \partial_l E_{ij} \end{cases} \tag{1.4}$$

by Lemma 3 in [17]. From the definition of F, we note that the assumption of $\det(E_0 + I) = 1$ is natural. The first two of these expressions are just the consequences of the incompressibility condition and the last one can be understood as the consistency condition for changing variables between the Lagrangian and Eulerian coordinates.

Recently, when ρ is a constant, system (1.1) has been studied extensively. Lin in [18], Lei in [17], Lin and Zhang in [19] proved the local wellposedness of (1.1) in Hilbert space H^s, and global wellposedness with small initial data. Here we mention that the key ingredient observation in [17] is that $\nabla \times F$ is a high order term for initial data under the physical considerations. Local wellposedness can be proved by the standard energy method, while to obtain a global result, a very subtle energy estimate is applied to capture the damping mechanism on $F - I$. When one adds a linear damping term in the evolution equation of FF^T, which is the Cauchy–Green strain tensor, Chemin and Masmoudi [7] proved the existence of a local solution and a global small solution in critical Besov spaces. We refer to [21] for the wellposedness of the system (1.1) in critical spaces. While in the L^p framework, T. Zhang and D. Fang proved the global existence of the strong solution for the incompressible viscoelastic fluid in [24].

In the present paper, we will consider the data with critical regularity in the sense of scaling invariance of the system. It is well known that such approach is well-oiled applied in the study of the wellposedness of incompressible Navier–Stokes equations (see [13,3,15,16] for details). The global wellposedness on inhomogeneous incompressible Navier–Stokes system was proved by Danchin in [10]. In the compressible case, which was considered by Danchin et al. in [9,4] and [8], though the system does not have any scaling invariance, authors proved the global wellposedness result in the critical space. Motivated by these work, D. Fang, B. Han and T. Zhang in [12] obtained the global existence in critical space for system (1.1) in the framework of L^2. The author also want to mention some significant works on the compressible viscoelastic fluids. J. Qian and Z. Zhang in [22] established the wellposedness of compressible viscoelastic fluid system in critical spaces. More precisely, the author in [22] proved the local existence result in the critical L^p framework and global wellposedness in the critical L^2 framework. The key point of the proof is to obtain the damping mechanism of the pressure P and the deformation tensor F. Recently, J. Jia, J. Peng and Z. Mei in [14]