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Long-time behavior of solution to the compressible magnetohydrodynamic equations

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ABSTRACT

In this paper, we study the long-time behavior of solution for the compressible magnetohydrodynamic equations in three-dimensional whole space. More precisely, we establish optimal decay rate for the higher-order spatial derivatives of solution, which improves the work of Li and Yu (2011), Chen and Tan (2010). Furthermore, one also investigates the long-time behavior for the mixed space–time derivatives of solution.

1. Introduction

In this paper, we consider the equations of magnetohydrodynamics which describe the motion of electrically conducting media in the presence of a magnetic field. The interaction between the viscous, isentropic, compressible fluid motion and the magnetic field are modeled by the magnetohydrodynamic system which describes the coupling between the compressible Navier–Stokes equations and the magnetic equations, i.e.,

$$\begin{cases} \rho_t + \operatorname{div}(\rho u) = 0, \\ (\rho u)_t + \operatorname{div}(\rho u \otimes u) - \mu \Delta u - (\mu + \lambda)\nabla \operatorname{div} u + \nabla P = (\operatorname{curl} B) \times B, \\ B_t - \operatorname{curl}(u \times B) = \nu \Delta B, \qquad \operatorname{div} B = 0, \end{cases} \tag{1.1}$$

where $(x,t) \in \mathbb{R}^3 \times \mathbb{R}^+$. Here the functions ρ, u, B and $P(\rho)$ represent the density, velocity, magnetic field and pressure respectively. The pressure $P(\rho)$ is a smooth function in a neighborhood of 1 with $P'(1) = 1$.

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The constants μ and λ denote the viscosity coefficients of the flow and satisfy

$$\mu > 0, \qquad 2\mu + 3\lambda \geq 0.$$

The positive constant ν is the magnetic diffusivity acting as a magnetic diffusion coefficient of the magnetic field. To complete the system (1.1), the initial data are given by

$$(\rho, u, B)(x,t)|_{t=0} = (\rho_0(x), u_0(x), B_0(x)). \tag{1.2}$$

Furthermore, as the space variable tends to infinity, we assume

$$\lim_{|x|\to\infty} (\rho_0 - 1, u_0, B_0)(x) = 0. \tag{1.3}$$

Due to their physical importance, mathematical complexity and wide range of applications, there are many studies on the MHD equations. In the one-dimensional case, the compressible MHD equations have been studied in many papers [2–4,9,15] and the references therein. For the two-dimensional case, Kawashima [14] obtained the global existence of smooth solution to the general electromagnetic fluid equations when the initial data are small perturbations of some given constant state. For the three-dimensional compressible MHD equations, Umeda et al. [20] obtained the global existence and the decay rate of smooth solution to the linearized MHD equations. The local strong solution to the nonlinear compressible MHD equations was obtained by Fan and Yu [6] in the presence of vacuum. Recently, Hu and Wang [10,12] and Fan and Yu [5] established the global weak solutions to the nonlinear compressible MHD equations with general initial data respectively. As is well known, for these weak and variational solutions, the uniqueness is an open problem. The low-Mach-limit problem for isentropic MHD equations was studied by Hu and Wang [11] and Jiang et al. [13] for different cases.

Recently, the study of decay rate for solution to the MHD equations has aroused many researchers' interest. First of all, under the H^3-framework, Li and Yu [16] and Chen and Tan [1] not only established the global existence of solution, but also obtained the decay rate of solution for the three-dimensional compressible MHD equations by assuming the initial data belong to L^1 and $L^q(q \in [1, \frac{6}{5}))$ respectively. More precisely, Chen and Tan [1] built the decay rate as follows

$$\|\nabla^k(\rho - 1, u, B)(t)\|_{H^{3-k}} \lesssim (1+t)^{-\frac{3}{2}\left(\frac{1}{q}-\frac{1}{2}\right)-\frac{k}{2}}, \tag{1.4}$$

where $k = 0, 1$. The decay rate (1.4) has also been established by Li and Yu [16] for the case $q = 1$. Motivated by the work of Guo and Wang [8], Tan and Wang [19] established the optimal decay rate for the higher-order spatial derivatives of solution if the initial data belong to $H^N \cap \dot{H}^{-s}$, where $N \geq 3$ and $s \in [0, \frac{3}{2})$. More precisely, they built the following decay rate

$$\|\nabla^k(\rho - 1, u, B)(t)\|_{H^{N-k}} \lesssim (1+t)^{-\frac{k+s}{2}}, \tag{1.5}$$

where $k = 0, 1, \ldots, N-1$. It is easy to see that the decay rate (1.5) provides faster time decay rate for the higher-order spatial derivatives of solution.

In this paper, one applies the Fourier splitting method by Schonbek [18] to establish faster decay rate for the higher-order spatial derivatives of solution to the compressible magnetohydrodynamic equations (1.1)–(1.3) if the initial data belong to $H^3 \cap L^q(q \in [1, \frac{6}{5}))$. Furthermore, we build the decay rate for the mixed space–time derivatives of solution.

Notation. In this paper, we use $H^s(\mathbb{R}^3)(s \in \mathbb{R})$ to denote the usual Sobolev spaces with norm $\|\cdot\|_{H^s}$ and $L^p(\mathbb{R}^3)(1 \leq p \leq \infty)$ to denote the usual L^p spaces with norm $\|\cdot\|_{L^p}$. The symbol ∇^l with an integer $l \geq 0$ stands for the usual any spatial derivatives of order l. We also denote $\mathscr{F}(f) := \hat{f}$. The notation $a \lesssim b$ means that $a \leq Cb$ for a universal constant $C > 0$ independent of time t. The notation $a \approx b$ means $a \lesssim b$ and $b \lesssim a$. For the sake of simplicity, we write $\|(A, B)\|_X = \|A\|_X + \|B\|_X$ and $\int f dx := \int_{\mathbb{R}^3} f dx$.