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Dynamics of second order in time evolution equations with state-dependent delay

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We deal with a class of second order in time nonlinear evolution equations with state-dependent delay. This class covers several important PDE models arising in the theory of nonlinear plates. We first prove well-posedness in a certain space of functions which are C^1 in time. In contrast with the first order models with discrete state-dependent delay this result does not require any compatibility conditions. The solutions constructed generate a dynamical system in a C^1-type space over delay time interval. Our main result shows that this dynamical system possesses compact global and exponential attractors of finite fractal dimension. To obtain this result we adapt the recently developed method of quasi-stability estimates.

1. Introduction

Our main goal is to study well-posedness and asymptotic dynamics of second order in time equations with delay of the form

$$\ddot{u}(t) + k\dot{u}(t) + Au(t) + F(u(t)) + M(u_t) = 0, \quad t > 0, \tag{1}$$

in some Hilbert space H. Here the dot over an element means time derivative, A is linear and $F(\cdot)$ is nonlinear operators, $M(u_t)$ represents (nonlinear) delay effect in the dynamics. All these objects will be specified later.

The main model we keep in mind is a nonlinear plate equation of the form

$$\partial_{tt}u(t,x) + k\partial_t u(t,x) + \Delta^2 u(t,x) + F(u(t,x)) + au(t - \tau[u(t)], x) = 0, \quad x \in \Omega,\ t > 0, \tag{2}$$

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in a smooth bounded domain $\Omega \subset \mathbb{R}^2$ with some boundary conditions on $\partial\Omega$. Here τ is a mapping defined on solutions with values in some interval $[0, h]$, k and a are constants. We assume that the plate is placed on some foundation; the term $au(t - \tau[u(t)], x)$ models effect of the Winkler type foundation (see [40,44]) with delay response. The nonlinear force F can be Kirchhoff, Berger, or von Karman type (see Section 6.1). Our abstract model covers also wave equation with state-dependent delay (see the discussion in Section 6.2).

We note that plate equations with *linear* delay terms were studied before mainly in Hilbert L_2-type spaces on delay interval (see, e.g., [2,3,10,11] and the references therein). However this L_2-type situation does not cover satisfactory the case of a state-dependent delay of the form described above. The point is that in this case the delay term in (2) is not even locally Lipschitz and thus difficulties related to uniqueness may arise. The desire to have Lipschitz property for this type of delay terms leads naturally to C-type spaces which are not even reflexive. This provides us with additional difficulties in contrast with the general theory well-developed for second order in time equations in the Hilbert space setting, see, e.g., [7] and also the literature cited there. In particular, in contrast with the non-delayed case (see [7–9]), in order to prove asymptotic smoothness of the flow (it is required for the existence of a global attractor) we are enforced to assume that the nonlinearity F is either subcritical (in the sense of [7]) or else the damping coefficient k in (1) is large enough. The main reason for this is that we are not able to apply Khanmamedov's or Ball's methods (see a discussion of both methods and more references in [9]). The point is that we cannot guarantee uniform in t weak continuity in the phase space of the corresponding functionals. Another reason is that the delay term destroys the gradient structure of the model in the case of potential nonlinearities F.

The studies of state-dependent delay models have a long history. As it is mentioned in [23] early discussion of differential equations with such a delay goes back to 1806 when Poisson studied a geometrical problem.[1] Since that time many problems, initially described by differential equations without delay or with constant delay, have been reformulated as equations with state-dependent delay. It seems rather natural because many models describing real world phenomena depend on the past states of the system. Moreover, it appears that in many problems the constancy of the time delay is just an extra assumption which makes the study easier. The waiver of this assumption is naturally lead to more realistic models and simultaneously makes analysis more difficult. The general theory of (ordinary) differential equations with state-dependent delay has been developed only recently (see, e.g., [25,31,45] and also the survey [23] and the references therein). This theory essentially differs from that of constant or time-dependent delays (see the references above and also Remark 2.1).

As for partial differential equations (PDEs) with delay their investigation requires the combination of both theories, methods and machineries (PDEs and delayed ODEs). The general theory of delayed PDEs was started with [19,43] at the abstract level and was developed in last decades mainly for parabolic type models with constant and time-dependent delays (see, e.g., the monographs [47] and the survey [39]). Abstract approaches for C-type [19,43] and L_p-type [26] phase spaces are available. We also mention a recent strong activity on dissipative PDEs with infinite delay which has the form of convolution in time with the proper kernel (see, e.g., [13,15,33] and the references therein).

Partial differential equations with state-dependent delay are essentially less investigated, see the discussion in the papers [12,34,35] devoted to the parabolic case. Some results (mainly, the existence and uniqueness) for the second order in time PDEs with delay are available. Most of them are based on a reformulation of the problem as a first order system and application of the theory of such systems (see, e.g., [19]). We also use this idea to get a local existence and uniqueness for problem (1). It is also worth mentioning the papers [20,24] which involve the theory of m-accretive (see [41], for instance) operators. However to the best of our knowledge, global and asymptotic dynamics of second order in time partial differential equations with state-dependent delay have not been studied before.

[1] We refer to [46] for a modern and detailed discussion of Poisson's example with state-dependent damping.