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Concentration and cavitation in the Euler equations for nonisentropic fluids with the flux approximation☆

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ABSTRACT

In this paper, by introducing a two-parameter flux approximation including pressure in the Euler equations for nonisentropic compressible fluids, we analyze the phenomena of concentration and cavitation in Riemann solutions. It is rigorously proved that, as the double parameter flux perturbation vanishes, any Riemann solution containing two shock waves and a possibly one-contact-discontinuity tends to a delta shock solution to the transport equations, and the intermediate density between the two shocks tends to a weighted δ-measure which forms the delta shock wave; any Riemann solution containing two rarefaction waves and a possibly one-contact-discontinuity tends to a two-contact-discontinuity solution to the transport equations, and the nonvacuum intermediate state in between tends to a vacuum state. Some numerical results are also given to present the processes of concentration and cavitation as the flux perturbation decreases.

1. Introduction

The full Euler equations in Eulerian coordinates read

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + \left(\rho u^2 + P\right)_x = 0, \\ (\rho E)_t + \left((\rho E + P)u\right)_x = 0, \end{cases} \tag{1.1}$$

which model the nonisentropic compressible fluids, where ρ is the density, u the velocity, ρE the total energy and P the scalar pressure. ρ and u are in a physical region $\{(\rho, u)|\rho \geq 0, |u| \leq U_0\}$ for some $U_0 > 0$. The

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pressure P is a function of ρ, u and ρE for polytropic gases:

$$P = (\gamma - 1)\left(\rho E - \frac{\rho u^2}{2}\right). \tag{1.2}$$

The Riemann problem and Cauchy problem of (1.1) were studied in [6,9,22,23].

In (1.1), when the pressure vanishes, the system formally becomes the transport equations

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2)_x = 0, \end{cases} \tag{1.3}$$

with an additional conservation law

$$(\rho E)_t + (\rho u E)_x = 0. \tag{1.4}$$

It is well-known that the transport equations, which are also called zero-pressure gas dynamics or Euler equations for pressureless fluids, are one of the most important models to study delta shock waves. It has attracted more and more attention since 1994, for example, see [1,2,10,11,17,18,20,27,29] and the references cited therein. Especially, Bouchut [1] first established the existence of measure solutions of the Riemann problem. E, Rykov and Sinai [10] studied the existence of global weak solution and the behavior of such global solution with random initial data. Sheng and Zhang [20] solved the 1-D and 2-D Riemann problems with the characteristic analysis and the vanishing viscosity method, see also [18]. Huang and Wang [11] obtained the uniqueness result of weak solution when the initial data is a Radon measure. It has been proved that δ-shock waves and vacuum states do occur in solutions to the transport equations (1.3). For δ-shock waves, we refer to the papers [12,13,15,24–26,31,32] for more details.

As stated in [8], fluids are substances whose molecular structure offers no resistance to external shear forces: even the smallest force causes deformation of a fluid particle. In most cases of interest, a fluid can be regarded as continuum, thus satisfies the balance laws of density, momentum, and energy. Fluids with large Mach number and far from solid surfaces can be described as the inviscid Euler equations. Thus, based on the numerical observation in [3–5] for the inviscid Euler equations of gas dynamics in the regime of small pressure: for one case, the particles seem to be more sticky and tend to concentrate at some shock locations which move with the associated shock speeds, and for the other case, the particles seem to be far apart and tend to form cavitation in the region of rarefaction waves. By introducing a small scaling parameter $\epsilon > 0$ modeling the strength of pressure P, that is,

$$P = \epsilon p = \epsilon(\gamma - 1)\left(\rho E - \frac{\rho u^2}{2}\right), \tag{1.5}$$

Chen and Liu [8] justified rigorously that, in inviscid nonisentropic flow, the phenomena of concentration and cavitation in the solutions are fundamental and the additional conservation law (1.4) yields the entropy inequality

$$(\rho u^2)_t + (\rho u^3)_x \leq 0 \tag{1.6}$$

in the sense of distributions for the Riemann solutions to the transport equations. In addition, they also rigorously analyzed the phenomena for isentropic fluids by looking at the vanishing pressure limit in [7]. Particularly, Li [16] investigated the zero temperature limit of solutions to the Euler equations for the isothermal case. Then, the results were extended to the relativistic fluid dynamics by Yin and Sheng [33], the perturbed Aw-Rascle model by Shen and Sun [19], the modified Chaplygin gas equations [30], etc. All these works above are only focused on the pressure level. This is one hand.

On the other hand, the small external shear forces imposed on the fluids lead to deformation of a fluid particle. Mathematically, the small forces can be regarded as a flux perturbation in terms of mechanics.