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Variational approach to homogenization of doubly-nonlinear flow in a periodic structure

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ABSTRACT

This work deals with the homogenization of an initial- and boundary-value problem for the doubly-nonlinear system

$$D_t w - \nabla \cdot \vec{z} = g(x, t, x/\varepsilon) \tag{0.1}$$

$$w \in \alpha(u, x/\varepsilon) \tag{0.2}$$

$$\vec{z} \in \vec{\gamma}(\nabla u, x/\varepsilon). \tag{0.3}$$

Here ε is a positive parameter; α and $\vec{\gamma}$ are maximal monotone with respect to the first variable and periodic with respect to the second one. The inclusions (0.2) and (0.3) are here formulated as *null-minimization principles*, via the theory of Fitzpatrick [MR 1009594]. As $\varepsilon \to 0$, a two-scale formulation is derived via Nguetseng's notion of two-scale convergence, and a (single-scale) homogenized problem is then retrieved.

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1. Introduction

This paper deals with the homogenization of a class of doubly-nonlinear parabolic equations of the form

$$\begin{cases} D_t w_\varepsilon - \nabla \cdot \vec{z}_\varepsilon = g(x, t, x/\varepsilon) \\ w_\varepsilon \in \alpha(u_\varepsilon, x/\varepsilon) \\ \vec{z}_\varepsilon \in \vec{\gamma}(\nabla u_\varepsilon, x/\varepsilon) \end{cases} \quad \text{in } \Omega \times]0, T[. \tag{1.1}$$

Here Ω is a bounded domain of $\mathbb{R}^N$, $T > 0$, and ε is a positive parameter. The (possibly multivalued) prescribed mappings

$$\alpha : \mathbb{R} \times \mathbb{R}^N \to \mathcal{P}(\mathbb{R}), \qquad \vec{\gamma} : \mathbb{R}^N \times \mathbb{R}^N \to \mathcal{P}(\mathbb{R}^N) \tag{1.2}$$

are assumed to be maximal monotone with respect to the first variable and periodic with respect to the second one. (By $\mathcal{P}$ we denote the set of the parts.) The known source field g is also periodic with respect to the third argument. We also assume that

$$\begin{aligned} &u_\varepsilon = 0 \quad \text{on } \partial\Omega \times]0, T[, \\ &w_\varepsilon(x, 0) = w^0(x, x/\varepsilon) \quad \text{for } x \in \Omega, \end{aligned} \tag{1.3}$$

for a prescribed periodic function w^0. All periods are assumed to coincide.

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Problems of the form (1.1) arise in several physical contexts: e.g., this may represent the entropy balance in diffusion phenomena; α may be the subdifferential of a dissipation potential. Existence of a solution for an associated boundary- and initial-value problem was proved e.g. by DiBenedetto and Showalter [8] and by Alt and Luckhaus [2].

In the case of single-valued operators, the homogenization of a system similar to (1.1) was already studied by H. Jian [13]. This was also used to model filtration in porous media by A.K. Nandakumaran and M. Rajesh [15,17,16]. More precisely, in [15] a quasi-linear equation of the form

$$\partial_t \alpha(u_\varepsilon, x/\varepsilon) - \nabla \cdot \vec{\gamma}(u_\varepsilon, \nabla u_\varepsilon, x/\varepsilon, t/\varepsilon) = g(x,t) \tag{1.4}$$

was studied with appropriate boundary and initial conditions, thus also accounting for high-frequency oscillations with respect to time. The same equation was also addressed by A. K. Nandakumaran and M. Rajesh [17,16], dealing with a porous medium with Neumann and Dirichlet boundary conditions, respectively. In [15,17] two-scale convergence was used extensively. It should be noticed that the Dirichlet condition on the boundary of the holes may yield different homogenized problems, that depend on the asymptotic relation between the size of the holes and the period ε.

The homogenization of quasi-linear equations has been studied by various authors, see e.g. [3,4,11,20]. The homogenization of doubly-nonlinear equations of the form (1.1) occurring in electromagnetic processes in composites and in Stefan-type problems was performed in [22,24].

Each of the inclusions $(1.1)_2$ and $(1.1)_3$ is equivalent to a variational inequality. On the basis of the Fitzpatrick theory [10], here we convert the system (1.1) to a linear PDE coupled with a *null-minimization* problem, along the lines of [26]. We then study the limit behavior for vanishing ε.

This note is organized as follows. First in Section 2 we briefly outline the Fitzpatrick theory for the variational representation of maximal monotone operators. In Section 3 we describe the homogenization problem to be studied, and reformulate it via the Brezis–Ekeland–Nayroles approach, see Problem 3.2. In Section 4 we prove existence of a solution via time-discretization, a priori estimates and passage to the limit, see Theorem 4.3. We then let ε vanish; in Section 5 we formulate the two-scale Problem 5.1, and in Theorem 5.4 we prove two-scale convergence to a solution of that problem. In Section 6 we then formulate the single-scale Problem 5.2, and in Proposition 6.4 we prove that it is equivalent to the two-scale problem. In Theorem 6.5 we then state the desired homogenization theorem. Finally, in an Appendix we briefly review Nguetseng's theory of two-scale convergence and related properties of integral functionals; these also include a result in preparation on the homogenization of maximal monotone operators.

The novelty of this work stays in the use of a Fitzpatrick-type formulation for homogenization, and in the derivation of a two-scale problem as an intermediate step towards homogenization.

The results of this note may be extended in several directions; for instance explicit dependence on time may be assumed in the nonlinear operator, and time-homogenization may also be considered. The homogenization of several other quasilinear equations may also be studied, including doubly-nonlinear systems of the form

$$w_\varepsilon - \nabla \cdot \vec{z}_\varepsilon = g(x,t,x/\varepsilon) \tag{1.5}$$

$$w_\varepsilon \in \alpha(D_t u_\varepsilon, x/\varepsilon) \tag{1.6}$$

$$\vec{z}_\varepsilon \in \vec{\gamma}(\nabla u_\varepsilon, x/\varepsilon), \tag{1.7}$$

with α and $\vec{\gamma}$ as above. Existence of a solution for an associated boundary- and initial-value problem was proved in [7].

2. Preliminaries

In this section we illustrate the tenets of the Fitzpatrick theory on the variational representation of maximal monotone operators, that is at the basis of the procedures of the present work. We also illustrate an idea of Brezis, Ekeland and Nayroles for the variational formulation of monotone flows. We refer the reader e.g. to [27] for a more detailed review.

2.1. Variational representation of maximal monotone operators

Let us first recall the Fenchel system, which is a basic result of the theory of convex analysis, see e.g. [9,21]. Let V be a separable and reflexive real Banach space with dual V', let $\psi : V \to \mathbb{R} \cup \{+\infty\}$ be a convex and lower semicontinuous function, and $\psi^* : V' \to \mathbb{R} \cup \{+\infty\}$ be its conjugate function, namely,

$$\psi^*(v') := \sup_{v \in V} \{\langle v', v\rangle - \psi(v)\} \quad \forall v' \in V'. \tag{2.1}$$

It is known that ψ, ψ^* and the *subdifferential* $\partial\psi$ (see e.g. [9,21]), satisfy the following Fenchel system:

$$\begin{cases} \psi(v) + \psi^*(v') \geq \langle v', v\rangle & \forall (v, v') \in V \times V', \\ \psi(v) + \psi^*(v') = \langle v', v\rangle & \text{if and only if } v' \in \partial\psi(v). \end{cases} \tag{2.2}$$

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