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Global existence and asymptotic behavior of solution for the sixth order Boussinesq equation with damped term[☆]

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ABSTRACT

We investigate the small-data Cauchy problem for the sixth order Boussinesq equation with damped term. The decay estimates of solutions to the corresponding linear equation are given by using the dyadic decomposition and some properties of Bessel functions. Then, we prove the global existence and asymptotic of the small amplitude solution in the time-weighted Sobolev space by the contraction mapping principle.

1. Introduction and main results

In this paper, we consider the Cauchy problem for the sixth order Boussinesq type equation with hydrodynamical damped term

$$u_{tt} - \Delta u - \Delta u_{tt} + \Delta^2 u + \Delta^2 u_{tt} - \Delta u_t = \Delta f(u), \quad x \in \mathbb{R}^n,\ t > 0 \tag{1.1}$$

$$u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \qquad x \in \mathbb{R}^n \tag{1.2}$$

where $u = u(x,t)$ is an unknown function, $u_0(x)$ and $u_1(x)$ are given initial data, $u_t = \frac{\partial u}{\partial t}$, $u_{tt} = \frac{\partial^2 u}{\partial t^2}$, Δ is the Laplacian operator in $\mathbb{R}^n$, $f \in C^k(\mathbb{R})$ is a given nonlinear function with the uniform polynomial behavior of growth which satisfies

$$|f^l(\cdot)| \lesssim |\cdot|^{\alpha - l}, \quad 0 \le l \le k \le \alpha,\ \alpha > 1. \tag{1.3}$$

In 1872, the Boussinesq equation

$$u_{tt} - u_{xx} + \gamma u_{xxxx} = (u^2)_{xx}$$

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was derived by Boussinesq [4], to describe the propagation of small amplitude long waves on the surface of shallow water. This was the first to give a scientific explanation of the existence to solitary waves found by Scott Russell's [18]. In order to investigate the water wave problem with surface tension, Schneider and Eugene [17] considered the following wave equation which models the water wave problem with surface tension

$$u_{tt} - u_{xx} - u_{xxtt} + \mu u_{xxxx} + u_{xxxxtt} = (u^2)_{xx}, \tag{1.4}$$

where $x, t, \mu \in \mathbb{R}$ and $u(x,t) \in \mathbb{R}$. The model can also be formally derived from the two-dimensional water wave problem. Eq. (1.4) with $\mu < 0$ is known as the bad Boussinesq equation because of its linear instability. For a degenerate case, Schneider and Eugene proved that the long-wave limit can be described approximately by the two decoupled Kawahara equations. Blow-up and scattering of solution for the Cauchy problem of equation

$$u_{tt} - u_{xx} - u_{xxtt} + u_{xxxx} + u_{xxxxtt} = f(u)_{xx} \tag{1.5}$$

are established in [22]. Wang and Xue [25] obtained the global existence and nonexistence of the Cauchy problem of Eq. (1.5) when $f(u) = \beta|u|^p$, $\beta \neq 0$ and $p > 1$ are constants, by the potential well method. In [21], Wang and Guo also studied the Cauchy problem for the Eq. (1.5). By using contracting mapping principle the authors proved the existence and uniqueness of the Boussinesq type equation. In addition, the sufficient conditions of blow-up of the solution for the problem in finite time were given. More recently, for the multidimensional cases of (1.5) and the special cases of nonlinear term like u^p, Suxia Xia and Jia Yuan established the global existence and uniqueness of the global small solutions as well as the small data scattering result to the Cauchy problem by the Littlewood–Paley dyadic decomposition [26].

So far, there are very few works on multi-dimensional Boussinesq type equation. Makhankov [13] pointed out the IBq (improved Boussinesq) equation

$$u_{tt} - \Delta u - \Delta u_{tt} = \Delta(u^2), \tag{1.6}$$

which can be obtained by starting with the exact hydro-dynamical set of equations in plasma, and a modification of the IBq equations analogous to the modified Korteweg–de Vries equation yields

$$u_{tt} - \Delta u - \Delta u_{tt} = \Delta(u^3). \tag{1.7}$$

Eq. (1.7) is the so-called IMBq (modified IBq) equation. Wang and Chen [20,19] studied the existence, both locally and globally in time, and nonexistence of solution, and the global existence of small amplitude solution for the Cauchy problem of the multidimensional generalized IMBq equation

$$u_{tt} - \Delta u - \Delta u_{tt} = \Delta f(u).$$

Posteriorly, by employing Besov spaces, some results of [19] were improved by Cho and Ozawa in [6,7].

Eq. (1.6) with one-dimension and its generalized form can describe the dynamical and thermodynamical properties of anharmonic monatomic and diatomic chains. To take into consideration internal friction (it is called this type of friction hydrodynamical), which is due to irreversible processes taking place within the system, the dissipation function depends on the time derivatives of the relative displacements, in [2] the authors obtained a damped Boussinesq (Bq) equation

$$u_{tt} - u_{xx} - u_{xxtt} - \nu u_{xxt} = f(u)_{xx}.$$

To take into account friction (it is called this type of external friction), which is due to the role of external medium on the particles taking place outside the system, the dissipation function depends on the particle velocity (Stokes law), in [2], the authors also obtained a Bq equation with Stokes damped

$$u_{tt} - u_{xx} - u_{xxtt} + \nu u_t = f(u)_{xx}.$$