Nonlinear Analysis 125 (2015) 457–467

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

The Dirichlet–Cauchy problem for nonlinear hyperbolic equations in a domain with edges

Vu Trong Luong*, Nguyen Thanh Tung

Department of Mathematics, Taybac University, Sonla, Viet Nam

ARTICLE INFO

Article history:
Received 25 February 2015
Accepted 12 May 2015
Communicated by Enzo Mitidieri

MSC:
35L70
35L20
35B65

Keywords:
Nonlinear hyperbolic equations
Regularity of solutions
Domain with edges

ABSTRACT

We study the Dirichlet–Cauchy problem for nonlinear hyperbolic equation of second order in a domain with edges. The aim of this paper is to prove the regularity of solution in weighted Sobolev spaces.

1. Introduction

Let Ω be a bounded domain in $\mathbb{R}^n$, $n > 2$, with the boundary $\partial\Omega$ consisting of two surfaces Γ_1, Γ_2 which intersect along a manifold l_0. Assume that in a neighborhood of each point of l_0 the set $\overline{\Omega}$ is diffeomorphic to a dihedral angle. For any $P \in l_0$, two half-spaces $T_1(P)$, and $T_2(P)$ tangent to Ω, and a two-dimensional plane $\pi(P)$ normal to l_0 are defined. We denote by $\nu(P)$ the angle in the plane $\pi(P)$ (on the side of Ω) bounded by the rays $R_1 = T_1(P) \cap \pi(P)$, $R_2 = T_2(P) \cap \pi(P)$ and by $\beta(P)$ the aperture of this angle. For each $0 < \tau \le T < \infty$, we set $Q_\tau = \Omega \times (0,\tau), S_\tau = \partial\Omega \times (0,\tau)$ (see Fig. 1).

Let L be a linear differential operator of second order on Q_T of the following form:

$$L(x,t,\partial)u = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}\Big(a_{ij}(x,t)\frac{\partial u}{\partial x_i}\Big) + \sum_{i=1}^{n} b_i(x,t)\frac{\partial u}{\partial x_i} + c(x,t)u, \tag{1.1}$$

where $a_{ij}(x,t), b_i(x,t), c(x,t)$ are real-valued functions on Q_T belonging to $C^{k+1}(Q_T)$. Throughout this paper, we assume that coefficients of L and its derivatives are bounded on Q_T. Moreover, suppose that

* Corresponding author.
E-mail addresses: vutrongluong@gmail.com (V.T. Luong), thanhtung70tbu@gmail.com (N.T. Tung).

http://dx.doi.org/10.1016/j.na.2015.05.028

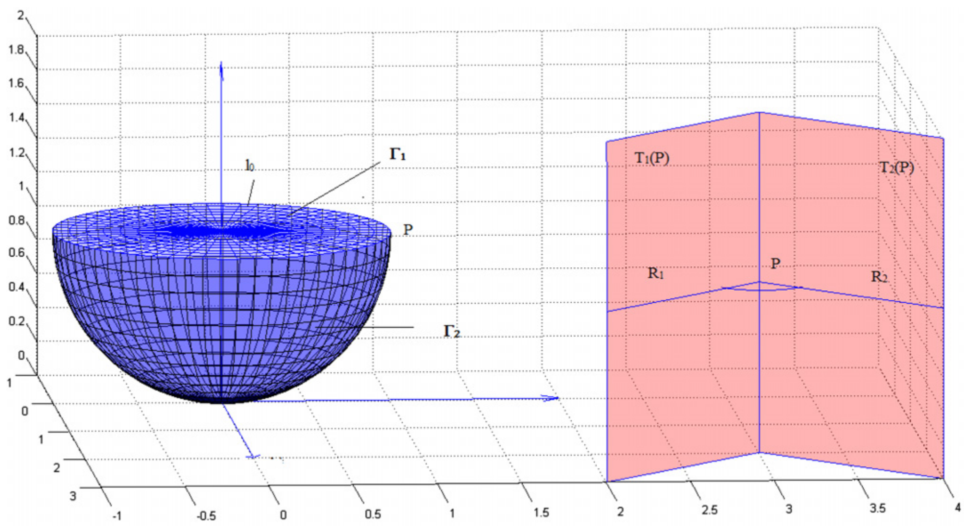

Fig. 1. A simple domain with edge.

$a_{ij} = a_{ji},\ (i,j = 1,\dots,n)$ are continuous in $x \in \overline{\Omega}$ uniformly with respect to $t \in [0,T]$ and

$$\sum_{i,j=1}^{n} a_{ij}(x,t)\xi_i\xi_j \geq \mu_0|\xi|^2 \tag{1.2}$$

for all $\xi \in \mathbb{R}^n \setminus \{0\}$ and $(x,t) \in Q_T$, where μ_0 is a positive constant.

For the operator L, we consider the following problem:

$$u_{tt} + L(x,t,\partial)u = f(x,t,u,Du) + h(x,t), \quad (x,t) \in Q_T, \tag{1.3}$$

$$u(x,0) = 0, \qquad u_t(x,0) = 0, \quad x \in \Omega, \tag{1.4}$$

$$u|_{S_T} = 0, \tag{1.5}$$

where f, h are given functions.

Initial boundary value problems (IBVPs) for nonlinear hyperbolic equations were studied in a number of papers. Most of these papers deal with the nonlinear wave equation. The existence or nonexistence of global solutions is investigated for example by J.L Lions [16], D.H. Sattinger [22,21], H.A. Levine [15], M. Can [2], B. Zheng [26] and many others. In [16], Lions considered the Cauchy–Dirichlet problem to semilinear wave equation by using the compactness method and Faedo–Galerkin techniques, the existence of solutions has been proved. In [22,21], Sattinger studied the existence and nonexistence of global solutions by the well potential method. By using this method, there have been a lot of results on qualitative analysis of nonlinear wave equations (see [4,25] and their references). The smoothness of solutions and the well-posedness of IBVPs for nonlinear hyperbolic equations and systems in a smooth domains have been investigated (see, e.g., [1,3,9–12,20,23,24]). Concerning the Dirichlet–Cauchy problem, we mention the papers of P. Brenner [1], A. Doktor [3], T. Kato[9,10], H. Koch [12], A. Milani and Y. Shibata [20]. The Neumann–Cauchy problem has been concerned in [11,23,24] by Y. Shibata. In [6] the existence, uniqueness and well-posedness for general class of quasilinear evolution equations on a short time interval are established by T.J.R. Hughes, T. Kato, and J.E. Marsden. These results are applied to quasilinear hyperbolic systems of second order on $\mathbb{R}^n$.

To the best of our knowledge, there are not many works published concerning the regularity of solutions of IBVPs in nonsmooth domains. For this issue, we mention some of papers which considered linear non-stationary equations in domains with conical points or with edges. In [14,7,8] linear parabolic/hyperbolic equations in domains with conical points were investigated in weighted Sobolev spaces in which the unique