Nonlinear Analysis 122 (2015) 215–238

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

The spreading frontiers in partially degenerate reaction–diffusion systems[☆]

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ARTICLE INFO

Article history:
Received 25 November 2014
Accepted 3 April 2015
Communicated by Enzo Mitidieri

MSC:
35K57
35B40
92D25

Keywords:
Spreading frontiers
Partially degenerate reaction–diffusion systems
Free boundary

ABSTRACT

In this paper, we study the spreading frontiers in the partially degenerate reaction–diffusion systems with a mobile and a stationary component, which are described by a class free boundary condition. The center of our investigation is to understand the effect of the dispersal rate D of the mobile component, the expansion capacity μ (the ratio of the expansion speed of the free boundary and the gradient of the mobile component at the spreading frontiers) and the initial numbers u_0 and v_0 on the long-run dynamics of the spreading frontiers. It is shown that a spreading–vanishing dichotomy holds, and the sharp criteria for the spreading and vanishing by choosing D, μ, u_0 and v_0 as variable factors are also obtained. In particular, our results still reveal that slow dispersal rate unconditionally favors the frontiers to spread, but the fast one, however, leads to a conditional vanishing of the frontiers. Moreover, as applications, we consider a man–environment–man epidemic model in physiology and a reaction–diffusion model with a quiescent stage in population dynamics.

1. Introduction

In the past decades, a wide variety of investigations on the non-degenerate reaction–diffusion systems have been obtained, where the word *non-degenerate* means that each diffusion coefficient in such systems is positive. On the other hand, there are still more and more phenomena in population biology, epidemiology and other disciplines that need to be modeled by partially degenerate reaction–diffusion systems, in which several diffusion coefficients are zeros. For example, more than four decades ago, Capasso and Maddalena [4] introduced an epidemic reaction–diffusion model described by the following coupled parabolic system to study the fecally–orally transmitted diseases in the European Mediterranean regions:

$$\begin{cases} u_t = Du_{xx} - a_{11}u + a_{12}v, \\ v_t = -a_{22}v + g_1(u), \end{cases} \tag{1.1}$$

☆ This work was supported by NSF of China (No. 11271172) and NSF of China (No. 11401473).

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http://dx.doi.org/10.1016/j.na.2015.04.003

for which [4] considered the threshold dynamics in a bounded domain, Zhao and Wang [34] established the existence of wavefronts and the minimal wave speed, and Wu [29] proved the existence of entire solutions in the bistable nonlinearity case, which behave like two monotone increasing traveling wave solutions propagating from both sides of the real line $\mathbb{R}$. To describe a single species population in which the individuals alternate between mobile and stationary states, Lewis and Schmitz [16] presented the following population model:

$$\begin{cases} u_t = Du_{xx} - \mu u - \gamma_1 u + \gamma_2 v, \\ v_t = \gamma v \left(1 - \dfrac{v}{K}\right) + \gamma_1 u - \gamma_2 v, \end{cases} \tag{1.2}$$

for which Hadeler and Lewis [13] obtained the existence of the spreading speed, and Wang and Zhao [25] considered the spreading speed and wavefronts by employing the general theory developed in [21]. Moreover, [13] proposed and discussed briefly the following similar population model:

$$\begin{cases} u_t = Du_{xx} + f_1(u) - \gamma_1 u + \gamma_2 v, \\ v_t = \gamma_1 u - \gamma_2 v. \end{cases} \tag{1.3}$$

For system (1.3), Zhang and Zhao [32] considered the asymptotic behavior of solutions in both unbounded and bounded domain, and Zhang and Li [31] further established the monotonicity and uniqueness of traveling wave solutions. Recently, Fang and Zhao [11] still studied the traveling fronts and spreading speed of the following general partially degenerate reaction–diffusion system:

$$\begin{cases} u_t = Du_{xx} + f(u, v), \\ v_t = g(u, v), \end{cases} \tag{1.4}$$

for which Wu et al. [30] further established the existence of entire solutions in the monostable nonlinearity case.

Although the study of traveling wave-like solutions in unbounded domain and threshold dynamics in bounded domain is an important issue of reaction–diffusion equations, it is not enough for the comprehensive mathematical understanding of the dynamical structure of solutions. In fact, either in unbounded domain or in bounded domain, the associated solutions are always positive for any $t > 0$, and eventually, converge to some positive equilibrium states no matter what the nonnegative nontrivial initial data are, which is just the well-known *hair-trigger effect* (cf. [2,3]). In realistic setting, for example, in population biology, this means that the population can spread immediately to whole environment even though there are only very rare population at the beginning, which does not match the realities that any population always spreads gradually and there are always the possibilities in which the population die out eventually. Moreover, there is another lack that little precise information about the spreading dynamics is presented.

In many applied areas, such as material science, biology, ecology, etc., the free boundary has been introduced to describe such a precise gradual spreading process, together with the changing of the underlying domain. In particular, there are more and more ecological models involved in the free boundary to characterize the spreading of species. For more details, we refer to [5–7,9,19,23,36] for single species population models, [8,12,17,22,24,27,28,35] for Lotka–Volterra systems, and [1,15] for epidemic models.

Inspired by the discussions above, the current research is devoted to a type of partially degenerate reaction–diffusion systems with a mobile component u and a stationary component v, in which the free boundary is introduced to describe the spreading frontiers of the mobile component. Specifically, we assume that two components both spread spatially to a gradual expanding domain in the left-end closed habitat $[0, \infty)$, but only the mobile one u can dispersal rightward, where the term “closed” means $u_x(0, t) = v_x(0, t) = 0$, i.e., there is not any flux at $x = 0$. The right spreading frontier is represented by a free boundary $h(t)$, which is lethal for both of the components. Assume that $h(t)$ grows at a rate proportional to the mobile component