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Existence and regularity results for doubly nonlinear inclusions with nonmonotone perturbation

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ABSTRACT

This paper deals with a class of nonlinear evolution inclusions arising from enthalpy formulation of heat conduction problems with phase change and nonmonotone source. Using time-discretization technique and monotone operators theory, we show the existence of weak solutions. Then we prove strong convergence of approximate solutions and lift regularity of the weak solutions under appropriate conditions. Moreover, application to physical model described by hemivariational inequality is given.

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1. Introduction

Let V and Z be separable and reflexive Banach spaces with the dual spaces V^* and Z^*, respectively. Let H be a Hilbert space which is identified with its dual H^* such that

$$V \subset Z \subset H \subset Z^* \subset V^*,$$

where all embeddings are dense and continuous and V embeds to Z compactly. Let $T > 0$, $1 < p < +\infty$ and q denote its conjugate exponent, i.e., $q = p/(p-1)$. Let $\mathcal{V}$ and $\mathcal{Z}$ stand for the vector-valued spaces $L^p(0,T;V)$ and $L^p(0,T;Z)$, respectively. Their dual spaces $L^q(0,T;V^*)$ and $L^q(0,T;Z^*)$ are denoted by $\mathcal{V}^*$ and $\mathcal{Z}^*$, respectively.

Under the above framework of spaces, we study the existence and regularity for nonlinear evolution inclusion in the following abstract form:

$$\frac{d}{dt}B(u(t)) + A(t,u(t)) + G(t,u(t)) \ni f(t) \quad \text{in } V^*, \ \text{a.e. } t \in (0,T), \tag{1.1}$$

where B is the subdifferential of a proper, convex and lower semicontinuous functional Ψ defined on Z, $A(t,\cdot): V \mapsto V^*$ is a pseudomonotone operator and $G(t,\cdot): Z \mapsto 2^{Z^*}$ is a nonmonotone perturbation. Both A and G depend on t explicitly.

In past several decades, increasing studies have been devoted to the doubly nonlinear inclusion in reflexive Banach spaces of the form:

$$\frac{d}{dt}B(u(t)) + A(u(t)) \ni f(t) \quad \text{a.e. } t \in (0,T), \tag{1.2}$$

which arises from a variety of diffusion problems such as non-stationary flow through a porous medium and heat transfer of solid–liquid phase change (cf. [4]). The study of (1.2) often requires B and A to be subdifferentials of convex functionals or

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maximal monotone operators, see, e.g., [11,2,15,1]. Two main methods, i.e., time-discretization and Yosida approximation, are developed to show the existence of solutions to (1.2). In this paper, we consider its nonmonotone perturbation problem (1.1) where A is pseudomonotone and G includes Clarke's generalized gradient as a special case.

As we know, pseudomonotone operators, introduced by Brézis [5], are significant generalizations of monotone operators. Operators of this type contain a variety of variational-type operators (cf. [18, Chapter 2]) and play an important role in the studies of nonlinear problems. In particular, a maximal monotone operator with its domain being the whole space is pseudomonotone, and a monotone and hemicontinuous operator is pseudomonotone. The concept of pseudomonotone operators was generalized by Browder and Hess [6] to multivalued case which has been widely applied to hemivariational inequality problems, cf. [26,7,8,19,20,23,25] etc.

It is easy to see that (1.1) contains parabolic inclusion as a special case. An earlier existence result for the parabolic inclusion follows from a classical surjectivity theorem developed by Papageorgiou et al. [28], which requires $L := \frac{d}{dt}B$ to be a linear maximal monotone operator. Then upper and lower solutions method was used to deal with the parabolic inclusion and related hemivariational inequalities by Carl, Le and Motreanu [8,7]. Recently, this problem has been considered by Kasyanov, Mel'nik and Toscano [14]. By using Faedo–Galerkin method, they derived the existence results under the assumption that $A+G$ is multivalued W_{λ_0} pseudomonotone (similar to generalized pseudomonotone on $D(L)$, cf. [29, Theorems 2, 3]). We would like to mention that Rothe method was also well designed to show the existence and regularity of parabolic variational or hemivariational inequalities (cf. Savaré [32,33], Kalita [12,13]). However, as far as we know, few papers have studied doubly inclusions (1.1) or (1.2) in which A is pseudomonotone.

By introducing a class of β-pseudomonotone operators, Maitre and Witomski dealt with (1.2) skillfully in [21]. The β-pseudomonotonicity of A, however, requires that B be single-valued, strictly increasing and the growth order of A be strictly lower than that of B in applications. Following this, we considered a class of hemivariational inequalities in [30]. Subsequently, the existence of weak solutions to boundary hemivariational inequalities with doubly nonlinear operators was studied in [31]. In this paper, we establish the existence and regularity of nonlinear inclusion (1.1) in quite a general framework. One of the potential applications of the results is the heat conduction problems with phase change and nonmonotone heat source. This work generalizes the existence results of parabolic problems such as [8,28,19,13,22,14] due to B being nonlinear and multivalued, and those of doubly nonlinear problems [1,2,16,15,4,11] in view of the pseudomonotonicity of A and the nonmonotone term G.

We would like to mention that the structure in the proof of weak solutions is similar to [30,31] since time-discretization method has been used in both of them. However, in this paper, new results have been obtained and some conclusions have been improved in comparison with the previous works. To be specific, the function spaces are handled with $p > 1$ instead of $p \geq 2$ as done in previous papers. Unlike [30], here B is multivalued, instead of being single-valued and strictly increasing, and the restriction of growth order on A and B has been removed. [31] is concerned with the weak solution of a class of boundary hemivariational inequalities, while this paper focuses on doubly nonlinear inclusions with a general nonmonotone term G which is time-dependent and includes the Clarke's generalized gradient as a special case. At the same time, the condition (B2) is nicer than that in [31] and the estimates in this paper are better than previous ones. Moreover, the relaxed coerciveness (G3) is removed in case of $p = 2$ (see Theorem 3.9). Finally, it is worth mentioning that in comparison with the previous works, this one studies strong convergence and regularity of the solutions and obtains some important results.

The plan of this paper is as follows: Section 2 is concerned with the preliminaries, basic hypotheses and the existence theorem of weak solutions to (1.1) (Theorem 2.10). This theorem is proved in Section 3 by time-discretization method. An existence theorem without the relaxed coerciveness of G is also established in this section (Theorem 3.9). Then, we study the strong convergence of the approximate solutions and the regularity of weak solutions in Section 4. In Section 5, we provide a physical model described by hemivariational inequality, to which our abstract results are applied. For the sake of conciseness, we prove several lemmas in the last section.

2. Preliminaries and weak solutions

Throughout this paper, by $\langle \cdot, \cdot \rangle_X$ we denote the duality pairing between a reflexive Banach space X and its dual X^*, by $\| \cdot \|_X$ the norm on X. When no confusion arises, the subscript X may be omitted for simplicity. We begin by reviewing some information on convex analysis and nonsmooth analysis (cf. [26, Chapter 1], [9, Chapter 2]).

Let $\Gamma_0(X)$ stand for the set of proper, convex and lower semicontinuous functionals defined on X. The subdifferential operator $\partial\phi : X \mapsto 2^{X^*}$ of $\phi \in \Gamma_0(X)$ is given by

$$\partial\phi(u) := \{w \in X^* : \phi(v) - \phi(u) \geq \langle w, v - u \rangle_X, \text{ for all } v \in D(\phi)\},$$

where $D(\phi) := \{u \in X : \phi(u) < +\infty\}$ denotes the effective domain. Furthermore, the convex conjugate of $\phi \in \Gamma_0(X)$ is defined as

$$\phi^*(v) := \sup_{u \in X}\{\langle v, u \rangle_X - \phi(u)\}, \quad \text{for all } v \in X^*.$$

As we know, ϕ^* belongs to $\Gamma_0(X^*)$, and for each pair $(u, w) \in X \times X^*$, the following are equivalent:

$$w \in \partial\phi(u), \quad \phi^*(w) = \langle w, u \rangle_X - \phi(u), \quad u \in \partial\phi^*(w). \tag{2.1}$$

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