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Global attractor for the generalized double dispersion equation[☆]

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ABSTRACT

The paper studies the existence of global attractor for the generalized double dispersion equation arising in elastic waveguide model $u_{tt} - \Delta u - \Delta u_{tt} + \Delta^2 u - \Delta u_t - \Delta g(u) = f(x)$. The main result is concerned with nonlinearities $g(u)$ with supercritical growth. In that case we construct a subclass $\mathbb{G}$ of the limit solutions and show that it has a weak global attractor. Especially, in non-supercritical case, the weak topology becomes strong, the further regularity of the global attractor is obtained and the exponential attractor is established in natural energy space.

1. Introduction

In this paper, we are concerned with the existence of global attractor for the generalized double dispersion equation arising in elastic waveguide model

$$u_{tt} - \Delta u - \Delta u_{tt} + \Delta^2 u - \Delta u_t - \Delta g(u) = f(x) \quad \text{in } \Omega \times \mathbb{R}^+, \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with the smooth boundary $\partial\Omega$, on which we consider either the hinged boundary condition

$$u|_{\partial\Omega} = \Delta u|_{\partial\Omega} = 0, \tag{1.2}$$

or the clamped boundary condition

$$u|_{\partial\Omega} = 0, \qquad \left.\frac{\partial u}{\partial \nu}\right|_{\partial\Omega} = 0, \tag{1.3}$$

where ν is the unit outward normal on $\partial\Omega$, and the initial condition

$$u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \quad x \in \Omega, \tag{1.4}$$

and the assumptions on $g(u)$ and f will be specified later.

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In the study of nonlinear wave propagation in elastic waveguide, on account of the energy exchange between the waveguide and the external medium through the lateral surfaces of the waveguide, Samsonov et al. [28,27] established the so-called cubic double dispersion equation

$$u_{tt} - u_{xx} = \frac{1}{4}(cu^3 + 6u^2 + au_{tt} - bu_{xx} + du_t)_{xx} \tag{1.5}$$

to describe the longitudinal displacement of the elastic rod. Here $a, b, c > 0$ and $d \neq 0$ are some constants depending on the Young modulus, the shearing modulus, density of the waveguide and the Poisson coefficient. Obviously, Eq. (1.1) includes (1.5) as its special case. There have been lots of research studies on the well-posedness, blowup, asymptotic behavior and other properties of solutions for both the IBVP and the IVP of the equation of type (1.1) (see [1,5,6,8,9,22–26,30–33] and references therein). While for the investigation on the global attractor to Eq. (1.1), one can see [14,15,34–36] and references therein.

Global attractor is a basic concept in the research studies of the asymptotic behavior of the dissipative system. From the physical point of view, the global attractor of the dissipative equation (1.1) represents the permanent regime that can be observed when the excitation starts from any point in natural energy space, and its dimension represents the number of degree of freedom of the related turbulent phenomenon and thus the level of complexity concerning the flow. All the information concerning the attractor and its dimension from the qualitative nature to the quantitative nature then yield valuable information concerning the flows that this physical system can generate. On the physical and numerical sides, this dimension gives one an idea of the number of parameters and the size of the computations needed in numerical simulations. However, the global attractor may possess an essential drawback, namely, the rate of attraction may be arbitrarily slow and it cannot be estimated in terms of physical parameters of the system under consideration. While the exponential attractor overcomes the drawback because not only it has finite fractal dimension but also its contractive rate is exponential and measurable in terms of the physical parameters.

Chueshov and Lasiecka [12,11] studied the longtime behavior of solutions to the Kirchhoff–Boussinesq plate equation

$$u_{tt} + ku_t + \Delta^2 u = div[f_0(\nabla u)] + \Delta[f_1(u)] - f_2(u) \tag{1.6}$$

with $\Omega \subset \mathbb{R}^2$ and the clamped boundary condition (1.3). Here $k > 0$ is the damping parameter, the mapping $f_0 : \mathbb{R}^2 \to \mathbb{R}^2$ and the smooth functions f_1 and f_2 represent (nonlinear) feedback forces acting upon the plate, in particular,

$$f_0(\nabla u) = |\nabla u|^2 \nabla u, \qquad f_1(u) = u^2 + u.$$

Ignoring both restoring force $f_0(\nabla u)$ and feedback force $f_2(u)$ and replacing the inertial term u_{tt} by ϵu_{tt}, with $\epsilon > 0$ (the relaxation time) sufficiently small, Eq. (1.6) becomes the modified Cahn–Hilliard equation

$$\epsilon u_{tt} + u_t - \Delta(-\Delta u + f(u)) = g, \tag{1.7}$$

which is proposed by Galenko et al. [16–18] to model rapid spinodal decomposition in non-equilibrium phase separation processes. Grasselli et al. [20,19,21] studied the well-posedness and the longtime dynamics of Eq. (1.7) in both 2D and 3D cases, with hinged boundary condition. They established the existence of the global and exponential attractor for $\epsilon = 1$ in 2D case, and for $\epsilon > 0$ sufficiently small in 3D case. Taking $\epsilon = 1$ in (1.7) or taking $f_0(\nabla u) = \nabla u, f_2 = 0$ in (1.6), and taking into account the inertial force represented by $-\Delta u_{tt}$ and replacing the weak damping u_t by a strong one $-\Delta u_t$, Eq. (1.1) arises.

In 1D case, Dai and Guo [14,15] established in phase space $E_2 = H^2 \cap H_0^1 \times H_0^1$ the finite dimensional global attractor for the IBVP of Eq. (1.1), with hinged boundary condition (1.2). For the multidimensional case, Yang [34] established in E_2 the global attractor provided that the growth exponent p of the nonlinearity $g(u)$ is subcritical, that is, $1 \leq p < \frac{N}{(N-2)^+}$, with $N \leq 5$, where and in the context $a^+ = max\{a, 0\}$. Under the similar assumptions the author [35] also discussed the existence of global attractor for Eq. (1.1) on $\mathbb{R}^N$. Here the growth exponent $\tilde{p} = \frac{N}{N-2}(N \geq 3)$ is called critical because one cannot get the uniqueness of weak solutions and cannot define the solution semigroup according to the traditional manner as $p > \tilde{p}$. When $p > \tilde{p}$, by introducing the trajectory dynamical system, which does not require the uniqueness of solutions and is developed by Chepyzhov and Vishik [10], Yang [36] established the so-called trajectory attractor but in the trajectory phase space equipped with weak* topology and not in natural energy space.

In order to establish the global attractor in the sense of strong topology in the case of supercritical nonlinearity and without the uniqueness of solutions, Ball [2,3] proposed the concept of generalized semiflows and use it (see [3]) to study the longtime dynamics of the semi-linear evolution equation

$$u_{tt} - \Delta u + \beta u_t + g(u) = 0, \tag{1.8}$$

on a bounded domain $\Omega \subset \mathbb{R}^N$ with Dirichlet boundary condition. In the supercritical nonlinearity case:

$$|g(s)| \leq C(1 + |s|^r), \tag{1.9}$$

with $r > \frac{N}{(N-2)^+}$, based on an unproved assumption that every weak solution satisfies the energy equation, he showed that the related generalized semiflow possesses in natural energy space a global attractor without the requirement for the uniqueness of weak solutions. But by now, to the best of our knowledge, the unproved assumption is still an open problem. Recently, Carvalho, Cholewa and Dlotko [7] proposed the concept of 'the subclass $\mathcal{LS}$ of the limit solutions' and proved that the corresponding subclass $\mathcal{LS}$ of Eq. (1.8) has a weak global attractor provided that $1 < r < \frac{N+2}{(N-2)^+}$.