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The existence of a pullback attractor for the three dimensional non-autonomous planetary geostrophic viscous equations of large-scale ocean circulation

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ABSTRACT

In this paper, we prove the existence of a pullback attractor in $H^2(\Omega)$ for the process $\{U(t,\tau)\}_{t\geq\tau}$ associated with the three dimensional non-autonomous planetary geostrophic viscous equations of large-scale ocean circulation by asymptotic a priori estimates.

1. Introduction

In this paper, we consider the regularity of the pullback attractor for the following three dimensional non-autonomous planetary geostrophic equations of large-scale ocean circulation (see [16,18])

$$\nabla p + f\vec{k} \times v + \epsilon L_1 v = 0, \tag{1.1}$$

$$\frac{\partial p}{\partial z} + T = 0, \tag{1.2}$$

$$\nabla \cdot v + \frac{\partial w}{\partial z} = 0, \tag{1.3}$$

$$\frac{\partial T}{\partial t} + v \cdot \nabla T + w\frac{\partial T}{\partial z} + L_2 T = Q \tag{1.4}$$

in the domain

$$\Omega = M \times (-h, 0) \subset \mathbb{R}^3,$$

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where $M \subset \mathbb{R}^2$ is a bounded domain with smooth boundary ∂M, $h > 0$ is the depth of the ocean. The unknown functions in the planetary geostrophic equations are the velocity field $(v, w) = (v_1, v_2, w) = (v_1(x, y, z, t), v_2(x, y, z, t), w(x, y, z, t))$, the pressure $p(x, y, z, t)$ and the temperature function $T(x, y, z, t)$. The given function $f = f_0(\beta + y)$ is the Coriolis parameter, $\epsilon > 0$ is a small constant, $Q(x, y, z, t)$ is a heat source.

Throughout this paper, we denote the two-dimensional horizontal gradient, Laplacian by ∇, Δ, respectively, and the operators L_1 and L_2 are given by

$$L_1 = -A_h\Delta - A_v\frac{\partial^2}{\partial z^2},$$
$$L_2 = -K_h\Delta - K_v\frac{\partial^2}{\partial z^2},$$

where $A_h,\ A_v$ are positive molecular viscosity constants and $K_h,\ K_v$ are positive conductivity constants. For simplicity, let Γ_u be the upper boundary of Ω, let Γ_l be the lateral boundary of Ω and let Γ_b be the bottom boundary of Ω, i.e.,

$$\Gamma_u = \{(x, y, z) \in \bar{\Omega} : z = 0\},$$
$$\Gamma_l = \{(x, y, z) \in \bar{\Omega} : (x, y) \in \partial M, -h \le z \le 0\},$$
$$\Gamma_b = \{(x, y, z) \in \bar{\Omega} : z = -h\}.$$

Eqs. (1.1)–(1.4) are subject to the following boundary conditions with wind-driven on the top surface, nonslip and non-flux on the side walls and bottom (see [12,18])

$$A_v\frac{\partial v}{\partial z}\bigg|_{\Gamma_u} = \mu, \qquad w|_{\Gamma_u} = 0, \qquad \left(K_v\frac{\partial T}{\partial z} + \alpha(T - T^*)\right)\bigg|_{\Gamma_u} = 0, \tag{1.5}$$

$$\frac{\partial v}{\partial z}\bigg|_{\Gamma_b} = 0, \qquad w|_{\Gamma_b} = 0, \qquad \frac{\partial T}{\partial z}\bigg|_{\Gamma_b} = 0, \tag{1.6}$$

$$v \cdot \vec{n}\big|_{\Gamma_l} = 0, \qquad \frac{\partial v}{\partial \vec{n}} \times \vec{n}\bigg|_{\Gamma_l} = 0, \qquad \frac{\partial T}{\partial \vec{n}}\bigg|_{\Gamma_l} = 0 \tag{1.7}$$

and initial condition

$$T(x, y, z, \tau) = T_\tau(x, y, z), \tag{1.8}$$

where $\mu(x, y)$ is the wind stress, $\vec{n}$ is the normal vector on Γ_l, $T^*(x, y)$ is the typical temperature of the top surface and α is a positive constant.

Due to the boundary conditions (1.5)–(1.7), it is natural to assume that T^* satisfies the compatibility boundary condition

$$\frac{\partial T^*}{\partial \vec{n}}\bigg|_{\partial M} = 0. \tag{1.9}$$

The planetary geostrophic equations are derived from the Boussinesq equations for the planetary scale ocean by using standard scale analysis (see [13,12,14,15,21]). During the past several decades, many authors have considered the well-posedness of weak solutions and strong solutions, the long-time behavior of solutions for the three dimensional autonomous planetary geostrophic viscous equations of large-scale ocean circulation (see [2,16,17]). In [16], the authors proved the existence of global (in time) weak solutions for the three dimensional autonomous planetary geostrophic viscous equations of large-scale ocean circulation under the assumptions $Q \in L^2(\Omega),\ \mu \in L^2(M),\ T^* \in L^2(M)$ and $T_0 \in L^2(\Omega)$. The existence of global strong solutions for the three dimensional autonomous planetary geostrophic viscous equations of large-scale ocean circulation was proved in [17] by using the fact that the maximum principle on T was established in [6] under the assumption $T_0 \in L^\infty(\Omega)$ or $T_0 \in H^2(\Omega)$. However, in [2], under stronger conditions than [16], namely $Q \in L^2(\Omega),\ \mu \in H_0^1(M),\ T^* \in H^2(M)$ and $T_0 \in L^2(\Omega)$, the authors proved the well-posedness of weak solutions for the three dimensional autonomous planetary geostrophic viscous equations of large-scale ocean circulation and obtained the existence of global strong solutions for the three dimensional autonomous planetary geostrophic viscous equations of large-scale ocean circulation under weaker conditions than [17], i.e., $Q \in H^1(\Omega),\ \mu \in H_0^1(M),\ T^* \in H^2(M)$ and $T_0 \in H^1(\Omega)$. Meanwhile, the authors proved the existence of a finite dimensional global attractor in $H \subset L^2(\Omega)$ for the three dimensional autonomous planetary geostrophic viscous equations of large-scale ocean circulation by the Sobolev compactness embedding theorem. In [23], the authors proved the existence of global attractors in V and $H^2(\Omega)$ for the three dimensional autonomous planetary geostrophic viscous equations of large-scale ocean circulation by the Sobolev compactness embedding theorem and asymptotic a priori estimates, respectively.