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An asymmetric superlinear elliptic problem at resonance

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ABSTRACT

In this article, we study the existence and multiplicity of solutions to the problem

$$\begin{cases} -\Delta u = g(x,u), & \text{in } \Omega; \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N (N \geqslant 2)$ with smooth boundary, and $g : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ is a differentiable function. We will assume that $g(x, s)$ has a resonant behavior for large negative values of s and that a Landesman–Lazer type condition is satisfied. We also assume that $g(x, s)$ is superlinear, but subcritical, for large positive values of s. We prove the existence and multiplicity of solutions for problem (1.1) by using minimax methods and infinite-dimensional Morse theory.

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1. Introduction

The goal of this paper is to study the existence and multiplicity of solutions of the following boundary value problem

$$\begin{cases} -\Delta u = g(x,u), & \text{in } \Omega; \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Δ denotes the N-dimensional Laplacian, $\Omega \subset \mathbb{R}^N$, for $N \geqslant 2$, is an open bounded set with a smooth boundary, $\partial\Omega$, and $g : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ a differentiable function. By a solution of (1.1) we mean a weak solution, i.e., a function $u \in H_0^1(\Omega)$ satisfying

$$\int_\Omega \nabla u \cdot \nabla v dx = \int_\Omega g(x,u) v dx, \tag{1.2}$$

for any $v \in H_0^1(\Omega)$, where $H_0^1(\Omega)$ is the Sobolev space obtained through completion of $C_c^\infty(\Omega)$ with respect to the metric induced by the norm

$$\|u\| = \left(\int_\Omega |\nabla u|^2 dx \right)^{\frac{1}{2}}, \quad \text{for all } u \in H_0^1(\Omega).$$

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Denote by $0 < \lambda_1 < \lambda_2 < \lambda_3 < \cdots$ the eigenvalues of the linear problem

$$\begin{cases} -\Delta u = \lambda u, & \text{in } \Omega; \\ u = 0, & \text{on } \partial\Omega. \end{cases} \tag{1.3}$$

Assume the following conditions on the nonlinearity g and its primitive $G(x,s) = \int_0^s g(x,\xi)d\xi$, for $x \in \overline{\Omega}$ and $s \in \mathbb{R}$, are satisfied:

(L_1) $g \in C^1(\overline{\Omega} \times \mathbb{R}, \mathbb{R})$ with $g(x,0) = 0$, $g_s'(x,0) = \frac{\partial g}{\partial s}(x,0) = \lambda_m$, $m \neq 1$.

(L_2) There exists a constant σ such that $1 \leqslant \sigma < (N+2)/(N-2)$ for $N \geqslant 3$, or $1 \leqslant \sigma < \infty$ for $N = 2$, and

$$\lim_{s\to+\infty} \frac{g(x,s)}{s^\sigma} = 0,$$

uniformly for a.e. $x \in \Omega$.

(L_3) There are constants $\mu > 2$ and $s_0 > 0$ such that

$$0 < \mu G(x,s) \leqslant s g(x,s), \quad \text{for } s \geqslant s_0 \text{ and } x \in \Omega.$$

(L_4) $\frac{2N\sigma}{N+2} < \mu$.

(L_5) $\lim_{s\to-\infty} [g(x,s) - \lambda_1 s] = g_{-\infty}(x)$, uniformly for a.e. $x \in \Omega$ with $g_{-\infty} \in L^\infty(\Omega)$ such that $|g_{-\infty}(x)| \leq M$, for all $x \in \Omega$, and for some constant $M > 0$.

(L_6) [Landesman–Lazer (LL) Condition]:

$$\int_\Omega g_{-\infty}(x)\varphi_1(x)dx > 0,$$

where φ_1 is a positive eigenfunction associated with the first eigenvalue of the $(-\Delta, H_0^1(\Omega))$, and $\|\varphi_1\| = 1$.

(L_7) There exists $s_- < 0$ such that

$$2G(x,s) - g(x,s)s \leqslant 0, \quad \text{for all } s \leqslant s_-.$$

Condition (L_3) yields super-quadratic growth for $G(x,s)$ for large positive values of s. Condition (L_5) implies that the problem (1.1) has a resonant behavior for large negative values of s, given that

$$\lim_{s\to-\infty} \frac{g(x,s)}{s} = \lambda_1,$$

as a consequence of (L_5), where λ_1 is the first eigenvalue of $-\Delta$ with Dirichlet boundary conditions in Ω.

The main results of this paper are the following:

Theorem 1.1. *Let (L_1)–(L_7) be satisfied. Then, problem (1.1) has a nontrivial solution.*

Theorem 1.2. *Suppose that g satisfies (L_1)–(L_7). Assume also that there exists $t_o > 0$ such that $g(x,t_o) = 0$ for all $x \in \Omega$. Then, problem (1.1) has at least three nontrivial solutions.*

Conditions (L_1)–(L_4) are similar to those imposed by de Figueiredo in [8] on a general class of superlinear elliptic problems of the type (1.1), with the additional hypothesis that $g_s'(x,0) = \lambda_m$, for $m \neq 1$, in (L_1). Condition (L_5) is a version of the condition

$$\lim_{s\to-\infty} \frac{g(x,s) - \lambda s}{|s|^\alpha} = 0,$$

where $\alpha = 0$ and $\lambda = \lambda_1$ in de Figueiredo [8]. This makes our problem into a resonant problem for large negative values of s. It is known in the literature on resonant problems that a solvability condition of the Landesman–Lazer type, for instance, is needed. In this paper, this condition is provided by (L_6). Condition (L_6) is similar to that used by Cuesta et al. [7] in their study of the problem

$$\begin{cases} -\Delta u = \lambda_1 u + (u^+)^p + f(x), & \text{in } \Omega; \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.4}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with a smooth boundary $\partial\Omega$, $N \geqslant 3$, and $f \neq 0$ is a function satisfying $f \in L^r(\Omega)$ for some $r > N$ and $1 < p < \frac{N+1}{N-1}$. They proved that the problem (1.4) has a solution in $W^{2,r}(\Omega) \cap H_0^1(\Omega)$ under the assumption

$$\int_\Omega f\varphi_1 < 0.$$

In this paper, we are also interested in studying a resonant-superlinear problem under the Landesman–Lazer type condition (L_6). In our case, because of condition (L_1), 0 is a degenerate critical point of the functional associated to problem (1.1). We

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