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Generating an arbitrarily large number of isolas in a superlinear indefinite problem☆

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ABSTRACT

This paper analyzes the effect of an asymmetric weight on the bifurcation diagrams relative to a class of superlinear indefinite problems which admit an arbitrarily high number of positive solutions for certain values of the parameters involved in their formulation. The main result is that the secondary bifurcations which occur in the symmetric case (see López-Gómez et al. (2014)) give rise, in the asymmetric case, to bounded components of solutions, whose number grows arbitrarily as the number of the solutions of the problem grows.

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1. Introduction

This paper analyzes the number and the topological structure of the components of the set of positive solutions of the one dimensional boundary value problem

$$\begin{cases} -u'' = \lambda u + a_b(t)u^p & \text{in } (0,1) \\ u(0) = u(1) = M \end{cases} \tag{1.1}$$

where $M \in (0, \infty]$, $p > 1$ and $\lambda < 0$ are constants, and $a_b(t)$ is the *asymmetric* piecewise constant function defined by

$$a_b(t) = \begin{cases} -c_0 & \text{if } t \in [0, \alpha] \\ b & \text{if } t \in (\alpha, 1-\alpha) \\ -c_1 & \text{if } t \in [1-\alpha, 1] \end{cases} \tag{1.2}$$

with $\alpha \in (0, 0.5)$, $b \geq 0$ and $0 < c_0 \neq c_1 > 0$. When $M = \infty$, the solutions of (1.1) are called *large (or explosive) solutions* of

$$-u'' = \lambda u + a_b(t)u^p$$

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in $(0, 1)$. In such case, the boundary conditions should be understood as $\lim_{t\downarrow 0} u(t) = \infty$ and $\lim_{t\uparrow 1} u(t) = +\infty$, or, shortly, as $u(0) = u(1) = +\infty$.

Problem (1.1) is of a great interest from the perspective of mathematical biology, as it provides us with the steady states of the parabolic counterpart of (1.1), which models the evolution of a species u inhabiting the polluted territory $(0, 1)$, since $\lambda < 0$, where the individuals of the population u compete among them in the region where $a_b(t) < 0$, i.e. in $(0, \alpha) \cup (1-\alpha, 1)$, while they cooperate in $(\alpha, 1-\alpha)$, where $a_b(t) > 0$. Consequently, it is natural to focus the attention on positive steady states. In the available literature, such problems are said to be of *superlinear indefinite* type. They have been widely investigated, e.g. in [1–12].

In the special case when $M = 0$ and $b \leq 0$, it is well known that $u = 0$ is the unique nonnegative solution of (1.1) and that it is a global attractor for the parabolic counterpart of (1.1) (see, e.g., [13]). This means that interspecific competition combined with a polluted habitat must drive the species to extinction. On the other hand, when $b > 0$ the model (1.1) admits, at least, a positive solution if $M = 0$ (see, e.g., [2]), and at least two if $M > 0$, as it has been recently shown in [14,15]. As a by-product, the cooperative effects really facilitate the permanence of the species, and hence they increase the complexity of the dynamics. Moreover, it is also known that the spatial heterogeneities of the weight function $a_b(t)$, measured by the number of simple zeros of $a_b(t)$ in the context of Problem (1.1), might provoke multiplicity of positive solutions, not only for $\lambda < 0$, but also for $\lambda \geq 0$ (see [6–8]). Rather astonishingly, even in the simplest case when $a_b(t)$ exhibits a single hump, it has been recently found in [11] that the complexity of the solution set of (1.1) in the symmetric case $c_0 = c_1$, using b as the main bifurcation parameter, increases arbitrarily as λ approximates $-\infty$, through an increasing series of twists of the branch bifurcating from the unique solution of Problem (1.1) for $b = 0$ and associated secondary bifurcations from it.

The purpose of this paper is to break the symmetry of the weight function $a_b(t)$, by taking $c_0 \neq c_1$, in order to analyze the fine topological structure of the set of positive solutions of (1.1). Our main result establishes that the unique (rather intricate) component constructed in [11], and computed in [15] for $c_0 = c_1$, splits into an arbitrary large number of compact components (isolas) as λ approximates $-\infty$, plus an additional unbounded component establishing a homotopy between the unique positive solution of (1.1) for $b = 0$, denoted by u_0, whose existence and uniqueness were proven, e.g., in [13], and the metasolution

$$m(t) := \begin{cases} \ell_{c_0}(t) & \text{if } t \in [0, \alpha) \\ +\infty & \text{if } t \in [\alpha, 1-\alpha] \\ \ell_{c_1}(1-t) & \text{if } t \in (1-\alpha, 1] \end{cases}$$

where, for each $c > 0$, $\ell_c(t)$ stands for the unique (large) solution of

$$\begin{cases} -u'' = \lambda u - cu^p & \text{in } (0, \alpha) \\ u(0) = M, \quad u(\alpha) = +\infty. \end{cases}$$

The emergence of an *arbitrarily large* number of *isolas* as λ approximates $-\infty$ is an extremely remarkable feature which has never been documented before in the specialized literature, though some mechanisms to generate single isolas, but not a series of them, are well known (see, e.g., [16–18] and the references there in).

To construct the solutions of (1.1), we first consider the sets Σ_0 and Σ_1 of all the positive solutions of the sublinear problems

$$\begin{cases} -u'' = \lambda u - c_0 u^p & \text{in } (0, \alpha) \\ u(0) = M \end{cases} \quad \text{and} \quad \begin{cases} -u'' = \lambda u - c_1 u^p & \text{in } (1-\alpha, 1) \\ u(1) = M \end{cases} \tag{1.3}$$

respectively, as well as the curves reached in the phase plane by all the positive solutions of these problems at times α and $(1-\alpha)$, i.e.

$$\Gamma_0 := \left\{ \big(u(\alpha), u'(\alpha)\big) : u \in \Sigma_0 \right\}, \qquad \Gamma_1 := \left\{ \big(u(1-\alpha), u'(1-\alpha)\big) : u \in \Sigma_1 \right\}.$$

Then, observe that each solution u_I of

$$\begin{cases} -u'' = \lambda u + bu^p & \text{in } (\alpha, 1-\alpha) \\ (u(\alpha), u'(\alpha)) \in \Gamma_0, \quad (u(1-\alpha), u'(1-\alpha)) \in \Gamma_1, \end{cases} \tag{1.4}$$

provides us with the following solution of (1.1)

$$u(t) := \begin{cases} u_L(t) & \text{if } t \in [0, \alpha) \\ u_I(t) & \text{if } t \in [\alpha, 1-\alpha] \\ u_R(t) & \text{if } t \in (1-\alpha, 1], \end{cases}$$

where u_L and u_R solve the problems

$$\begin{cases} -u'' = \lambda u - c_0 u^p & \text{in } (0, \alpha) \\ u(0) = M, \quad u(\alpha) = u_I(\alpha), \end{cases} \quad \text{and} \quad \begin{cases} -u'' = \lambda u - c_1 u^p & \text{in } (1-\alpha, 1) \\ u(1-\alpha) = u_I(1-\alpha), \quad u(1) = M, \end{cases}$$

respectively. Up to the best of our knowledge, this methodology comes from [12].

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