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Stochastic neutral evolution equations on Hilbert spaces with partially observed relaxed control and their necessary conditions of optimality

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ABSTRACT

In this paper we develop the necessary conditions of optimality for partially observed control problems governed by stochastic neutral evolution equations on Hilbert spaces driven by relaxed controls.

1. Introduction

Let E be a separable Hilbert space and consider the neutral stochastic evolution equation on E of the form:

$$d(x + g(t, x)) = Axdt + b(t, x, u_t)dt + \sigma(t, x, u_t)dW, \qquad x(0) = x_0 \in E, \quad t \in I \equiv [0, T],\ T < \infty, \tag{1}$$

where $g : I \times E \longrightarrow E$, A is the infinitesimal generator of an analytic semigroup on E, and $b : I \times E \times U \longrightarrow E$ and $\sigma : I \times E \times U \longrightarrow \mathcal{L}(H, E)$ are Borel measurable maps. Let $(\Omega, \mathcal{F}, \mathcal{F}_{t\in I}, P)$ be a complete probability space with the filtration $\mathcal{F}_t \subset \mathcal{F}$. The process $W = \{W(t), t \geq 0\}$ is an $\mathcal{F}_t$-adapted H valued Brownian motion with incremental covariance operator $Q \in \mathcal{L}_1(H)$, the space of nuclear operators on H. The set U is a compact Polish space. Let $M_0(U)$ denote the class of regular probability measures on $\mathcal{B}(U)$, the sigma algebra of Borel subsets of the set U. Let $L^a_\infty(I, M_0(U))$ denote the class of weak star measurable $\mathcal{G}_t \subset (\mathcal{F}_t)$-adapted random processes with values in the space of probability measures $M_0(U)$. We consider $\mathcal{U}_{ad} \subset L^a_\infty(I, M_0(U))$ as the set of admissible controls. The problem is to find a control that minimizes the cost functional

$$J(u) \equiv \mathbf{E}\left\{\int_I \ell(t, x(t), u_t)dt + \Phi(x(T))\right\}, \tag{2}$$

where $\ell : I \times E \times U \longrightarrow \bar{R}$ is a Borel measurable (extended) real valued function and $\Phi : E \longrightarrow \bar{R}$ is a Borel measurable map. Precise assumptions will be given later.

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Regular stochastic systems on Hilbert spaces have been studied extensively. The corresponding literature is large. We mention here the well known book by Da Prato and Zabczyk [1] and the extensive references therein. Neutral stochastic systems are sufficiently general to include the abstract evolution equations and evolution equations arising from parabolic and hyperbolic partial differential equations with nonhomogeneous boundary data. For details the reader is referred to [2], ([3], part.1) and ([4], Chapter 3) and the references therein. There is an abundance of literature on stochastic minimum (or maximum) principle for finite dimensional problems [5–9], see also the extensive references therein. In [5] the authors prove the maximum principle for a class of regular stochastic functional differential equations. In [7] the author proves the maximum principle of Pontryagin type for finite dimensional neutral stochastic systems generalizing the results of Peng [9]. In [6] we considered the question of existence of relaxed controls for finite dimensional regular stochastic systems driven by Brownian motion and the Lévy process and also presented necessary conditions of optimality. In [8] the authors prove the maximum principle for mean-field stochastic systems. In [10] we considered the question of existence of optimal feedback controls for a class of infinite dimensional stochastic systems with a very general class vector fields. Recently we have studied control problems for partially observed regular stochastic systems on infinite dimensional Hilbert spaces [11] and developed the necessary conditions of optimality. But to the best of knowledge of the author, there is no literature on the necessary conditions of optimality for stochastic neutral systems on infinite dimensional Hilbert spaces driven by relaxed controls. In a recent paper [12] we proved only the existence of optimal relaxed controls for many standard and nonstandard control problems for neutral stochastic systems. Here in this paper we focus on the necessary conditions of optimality of such systems.

The rest of the paper is organized as follows. In Section 2, we introduce some basic background materials and notations. In Section 3, we present some recent results on the questions of existence and regularity of mild solutions of neutral evolution equations, question of continuous dependence of solutions on relaxed controls (endowed with certain weak topology) and the questions of existence of optimal controls. In Section 4, we study necessary conditions of optimality; and in Section 5, we present two examples, one on a general partial differential equation with nonhomogeneous boundary conditions, and one on the popular linear quadratic Gaussian regulator problem (LQGR).

2. Basic materials and notations

All random processes mentioned below are based on the complete filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_{t\in I}, P)$ without further notice. Let $B^a_\infty(I, E) \subset L^a_{\infty,2}(I \times \Omega, E) = L^a_\infty(I, L_2(\Omega, E))$ denote the space of $\mathcal{F}_t$-adapted random processes with values in the Hilbert space E satisfying

$$\sup\{\mathbf{E}|x(t)|^2_E, t \in I\} < \infty.$$

The space $B^a_\infty(I, E)$, equipped with the norm topology $\sup\{\sqrt{\mathbf{E}|x(t)|^2_E}, t \in I\}$, is a closed subspace of the Banach space $L^a_{\infty,2}(I \times \Omega, E)$ and hence itself a Banach space.

The space of linear (not necessarily bounded) operators from H to E is denoted by $L(H, E)$ and the space of bounded operators is denoted by $\mathcal{L}(H, E)$. The space $\mathcal{L}(H, E)$ equipped with the strong operator topology τ_{so} is denoted by $\mathcal{L}_{so}(H, E) \equiv (\mathcal{L}(H, E), \tau_{so})$. It is well known that this is a locally convex sequentially complete topological vector space [13]. The space of nuclear operators on H will be denoted by $\mathcal{L}_1(H)$. Recall that the incremental covariance operator of the Wiener process W is given by $Q \in \mathcal{L}^+_1(H)$. Using this operator we construct the vector space $\mathcal{L}_{2,Q}(H, E)$ of linear operators from H to E with the scalar product $\langle T_1, T_2\rangle_Q \equiv Tr(T_1QT_2^*)$ for $T_1, T_2 \in L(H, E)$. Note that these operators are not required to be bounded. The norm of any $T \in \mathcal{L}_{2,Q}(H, E)$ is given by the square root of $|T|^2_Q \equiv \langle T, T\rangle_Q \equiv Tr(TQT^*)$. Completion of this vector space with respect to the above norm topology is a Hilbert space denoted by the same symbol $\mathcal{L}_{2,Q}(H, E)$. We are interested in the random processes taking values in the Hilbert space $\mathcal{L}_{2,Q}(H, E)$. Let $L^a_2(I, \mathcal{L}_{2,Q}(H, E))$ denote the space of $\mathcal{F}_t$-adapted $\mathcal{L}_{2,Q}(H, E)$ valued random processes. For any $\sigma \in L^a_2(I, \mathcal{L}_{2,Q}(H, E))$, we define the norm given by

$$\|\sigma\| \equiv \left(\mathbf{E}\int_I Tr(\sigma(t)Q\sigma^*(t))dt\right)^{1/2}.$$

Furnished with this norm topology, $L^a_2(I, \mathcal{L}_{2,Q}(H, E))$ is also a Hilbert space.

As regards control, first let us consider the space $M(U)$ of signed Borel measures defined on the Borel field of subsets of the set U, where U is a compact Polish space. Let $L^a_\infty(I, M(U))$ denote the space of $\mathcal{F}_t$ adapted random processes with values in the space of signed measures $M(U)$. With respect to the norm topology, given by $ess - sup\{|u_t|_{\tau v}, (t, \omega) \in I \times \Omega\}$, this is a Banach space. By $|\nu|_{\tau v}$ we mean the total variation of the measure ν on U. We are not interested in the norm topology. For controls, we consider weak star topology on $L^a_\infty(I, M(U))$ and for admissible controls we choose an appropriate subset $\mathcal{U}_{ad}$ (described later) of the set $L^a_\infty(I, M_0(U))$ where $M_0(U) \subset M(U)$ denotes the space of probability measures defined on the Borel subsets of U. This space is endowed with the weak star topology denoted by τ_w. It follows from Alaoglu's theorem that it is a weak star compact subset of $L^a_\infty(I, M(U))$. Let $(D, \geq)$ be a directed index set. With respect to the weak star topology, a net $u^\alpha \xrightarrow{\tau_w} u^o (\alpha \in D)$ if and only if for every $\varphi \in L^a_1(I, C(U))$

$$\mathbf{E}\int_I\int_U \varphi(t, \xi)u^\alpha_t(d\xi)dt \longrightarrow \mathbf{E}\int_I\int_U \varphi(t, \xi)u^o_t(d\xi)dt.$$