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Invasion traveling wave solutions of a predator–prey system

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ABSTRACT

This paper is concerned with the traveling wave solutions of a predator–prey system, in which intra-specific conflictions are involved. By using the comparison principle and the asymptotic speed of spreading, the existence, nonexistence and minimal wave speed of non-negative traveling wave solutions are established. Such a traveling wave solution can model the spatial–temporal process where the predator invades the territory of the prey and they eventually coexist.

1. Introduction

In population dynamics, one cornerstone is the following Lotka–Volterra system [1,2]

$$\begin{cases} \dfrac{du_1(t)}{dt} = r_1u_1(t)\left[1 - bu_2(t)\right], \\ \dfrac{du_2(t)}{dt} = r_2u_2(t)\left[-1 + fu_1(t)\right], \end{cases} \tag{1.1}$$

in which all the parameters are positive. It is clear that u_1 and u_2 are in the positions of the prey and the predator in population dynamics, respectively, and the biological sense of these parameters is obvious. Since this model has been widely studied and the results can be found in many textbooks, we do not list its mathematical properties again. From the viewpoint of mathematical biology, the evolution of the predator only depends on its death rate and the number of prey in (1.1). Moreover, $u_2 \equiv 0$ always satisfies the second equation, so the number of prey will be unlimited if the predator vanishes and $u_1(0) > 0$ holds. Due to the possible infinite number of prey (see, e.g., Murray [3, p. 87]), this model is not sufficiently precise to describe some evolutionary processes, and many types of predator–prey systems have been proposed and widely studied, see Malchow et al. [4] for some examples. In particular, if we assume for both the prey and the predator intra-actions with logistic growth, then we may obtain the following model (see Malchow et al. [4] and Pielou [5])

$$\begin{cases} \dfrac{du_1(t)}{dt} = r_1u_1(t)\left[1 - u_1(t) - bu_2(t)\right], \\ \dfrac{du_2(t)}{dt} = r_2u_2(t)\left[-1 - u_2(t) + fu_1(t)\right]. \end{cases} \tag{1.2}$$

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In Zhang and Feng [6], (1.2) was also illustrated by considering the intra-specific social confliction of individuals in (1.1). If $f > 1$ holds, then (1.2) has a positive equilibrium $K = (k_1, k_2)$ defined by

$$k_1 = \frac{b+1}{bf+1}, \qquad k_2 = \frac{f-1}{bf+1}.$$

It is evident that the coexistence steady state K is stable while the semitrivial equilibrium $(1, 0)$ is unstable, and the trivial steady state $(0, 0)$ is also unstable, see [6].

In this paper, we shall consider the temporal–spatial structure of the following reaction–diffusion system

$$\begin{cases} u_{1t}(x,t) = d_1 u_{1xx}(x,t) + r_1 u_1(x,t)\left[1 - u_1(x,t) - b u_2(x,t)\right], \\ u_{2t}(x,t) = d_2 u_{2xx}(x,t) + r_2 u_2(x,t)\left[-1 + f u_1(x,t) - u_2(x,t)\right], \end{cases} \tag{1.3}$$

where $x \in \mathbb{R}, t > 0, d_1 > 0, d_2 > 0$ are diffusion coefficients and the other parameters are the same as those in (1.2). In population dynamics, $u_1(x,t)(u_2(x,t))$ denotes the population density of the prey (predator) at time t at location x. By rescaling, it is sufficient to investigate

$$\begin{cases} u_{1t}(x,t) = d u_{1xx}(x,t) + r u_1(x,t)\left[1 - u_1(x,t) - b u_2(x,t)\right], & x \in \mathbb{R},\ t > 0, \\ u_{2t}(x,t) = u_{2xx}(x,t) + u_2(x,t)\left[-1 + f u_1(x,t) - u_2(x,t)\right], & x \in \mathbb{R},\ t > 0, \end{cases} \tag{1.4}$$

where $d > 0, r > 0$ are positive constants, and b, f are the same as those in (1.3).

For the spatial–temporal patterns of predator–prey systems, one important case is to model the predator being introduced into the habitat of the prey [7, Chapter 8], and some dynamical properties of the phenomena can be formulated by means of traveling wave solutions, see Fagan and Bishop [8], Owen and Lewis [9] for the modeling of population evolution at Mount St. Helens. For the sake of convenience, we first introduce the following definition of traveling wave solutions.

Definition 1.1. A traveling wave solution of (1.4) is a special entire solution defined for all $x, t \in \mathbb{R}$ and taking the following form

$$(u_1(x,t), u_2(x,t)) = (\phi_1(x+ct), \phi_2(x+ct)),$$

where $(\phi_1, \phi_2) \in C^2(\mathbb{R}, \mathbb{R}^2)$ is the wave profile that propagates through the one-dimensional spatial domain $\mathbb{R}$ at the constant wave speed $c > 0$.

Let $\xi = x + ct \in \mathbb{R}$, then (ϕ_1, ϕ_2) and $c > 0$ must satisfy

$$\begin{cases} d\phi_1''(\xi) - c\phi_1'(\xi) + r\phi_1(\xi)\left[1 - \phi_1(\xi) - b\phi_2(\xi)\right] = 0, \\ \phi_2''(\xi) - c\phi_2'(\xi) + \phi_2(\xi)\left[-1 + f\phi_1(\xi) - \phi_2(\xi)\right] = 0. \end{cases} \tag{1.5}$$

Recalling the biological background mentioned above, we also require that (ϕ_1, ϕ_2) satisfy the following asymptotic boundary conditions

$$\lim_{\xi \to -\infty} (\phi_1(\xi), \phi_2(\xi)) = (1, 0), \qquad \lim_{\xi \to \infty} (\phi_1(\xi), \phi_2(\xi)) = (k_1, k_2). \tag{1.6}$$

Note that $\xi = x + ct$, then the solutions of (1.5)–(1.6) formulate the following biological process: At any fixed location $x \in \mathbb{R}$, there was only the prey a long time before ($t \to -\infty, x + ct \to -\infty$), and the prey and the predator will coexist after a long time ($t \to \infty, x + ct \to \infty$). Of course, we also claim that only the positive solutions of (1.5) will be investigated due to its ecological sense, and the main purpose of this paper is the existence and nonexistence of positive solutions to (1.5) with (1.6).

In the literature, traveling wave solutions of parabolic systems have been widely studied since [10,11], and Volpert et al. [12] presented the existence, uniqueness and stability of traveling wave solutions of monotone systems. Liang and Zhao [13] and Fang and Zhao [14] established a general scheme on the existence of traveling wave solutions of monotone semiflows. For the traveling wave solutions of predator–prey systems, the study is very hard since these systems cannot generate monotone semiflows so it is difficult to use the theory mentioned above. The analysis of the phase plane can also be useful for the existence of traveling wave solutions, but the geometric structure in $\mathbb{R}^4$ will be very complex, see Huang et al. [15] for an example. In the past decades, some classical results have been established for the existence of traveling wave solutions of predator–prey systems by different methods including the shooting method, Conley index and fixed point theorem, see Dunbar [16–18], Gardner and Smoller [19], Gardner and Jones [20], Huang [21], Hsu et al. [22], Liang et al. [23], Lin et al. [24], Wang et al. [25] and references cited therein.

Recently, several papers studied the traveling wave solutions of two species predator–prey systems in which other species may also be present in the environment where predator and prey are present and the predator has a choice of which species to feed upon (see Malchow et al. [4]), that is the following reaction–diffusion system

$$\begin{cases} u_{1t}(x,t) = d u_{1xx}(x,t) + r u_1(x,t)[1 - u_1(x,t) - b_1 u_2(x,t)], & x \in \mathbb{R},\ t > 0, \\ u_{2t}(x,t) = u_{2xx}(x,t) + u_2(x,t)[1 - u_2(x,t) + b_2 u_1(x,t)], & x \in \mathbb{R},\ t > 0, \end{cases} \tag{1.7}$$