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Hyperbolic mean curvature flow with a forcing term: Evolution of plane curves

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ABSTRACT

In this paper we study the evolution of closed strictly convex plane curves moving by the hyperbolic mean curvature flow with a forcing term. It is shown that the flow admits a unique short-time smooth solution and the convexity of the curves is preserved during the evolution. When the forcing term is a negative constant, we prove the curves either converge to a point or a C^0 curve. For a positive constant forcing term, the flow has a unique smooth solution in any finite time and expands to infinity as t tends to infinity if the initial curvature is smaller than M, the flow will blow up in a finite time if the initial curvature is larger than M.

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1. Introduction

In this paper we study the closed convex evolving plane curves under the hyperbolic mean curvature flow with a forcing term. More precisely, we consider such an initial value problem

$$\begin{cases} \dfrac{\partial^2 F}{\partial t^2}(u,t) = (k(u,t) - h(u))\vec{N}(u,t) - \left(\dfrac{\partial^2 F}{\partial s \partial t}, \dfrac{\partial F}{\partial t}\right)\vec{T}, \quad \forall\,(u,t) \in S^1 \times [0,T), \\ F(u,0) = F_0(u), \\ \dfrac{\partial F}{\partial t}(u,0) = f(u)\vec{N}_0, \end{cases} \tag{1.1}$$

where k is the mean curvature, $h(u)$ is a forcing term, $\vec{N}$ is the unit inner normal at $F(u,t)$, F_0 stands for a smooth strictly convex closed curve, $f(u)$ and $\vec{N}_0$ are the initial velocity and inner normal vector of F_0, respectively, and with $\vec{T}$ denoting the unit tangent vector and s the arclength parameter.

This system is an initial value problem for a system of partial differential equations for F, which can be completely reduced to an initial value problem for a single partial differential equation for its support function. The latter equation is a hyperbolic Monge–Ampère equation. Our first result is the following local existence theorem for the initial value problem (1.1).

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Theorem 1.1 (*Local Existence and Uniqueness*). *Suppose that F_0 is a smooth strictly convex closed curve. Then there exist a positive T and a family of strictly convex closed curves $F(\cdot, t)$ with $t \in [0, T)$ such that $F(\cdot, t)$ satisfies (1.1), provided that $h(u)$ and $f(u)$ are smooth functions on S^1.*

The following theorem concerns smooth solution to the specialized flow equation

$$\frac{\partial^2 F}{\partial t^2}(u, t) = (k(u, t) - M)\vec{N}(u, t) - \left(\frac{\partial^2 F}{\partial s \partial t}, \frac{\partial F}{\partial t}\right)\vec{T}, \quad \forall\, (u, t) \in S^1 \times [0, T), \tag{1.2}$$

where the forcing term M is a non-zero constant.

Theorem 1.2. *Suppose that F_0 is a smooth strictly convex closed curve with the zero initial velocity, then for any non-zero constant M, we have*

Case 1. *If $M < 0$, the solution of (1.2) exists on a maximal time interval $[0, T)$ with $T < \infty$, and the solution $F(\cdot, t)$ either converges to a point or converges to a C^0 curve.*

Case 2. *If $M > 0$, $\max k_{t=0} \leq M$ and $k \neq M$ on F_0, then the solution of (1.2) exists in any finite time and $F(\cdot, t)$ will expand to infinity as $t \to \infty$.*

Case 3. *If $M < 0$, $\min k_{t=0} > M$, then the solution of (1.2) exists on a finite time interval $[0, T)$, and the solution $F(\cdot, t)$ either converges to a point or converges to a C^0 curve.*

Remark 1.1. The theorem not only indicates that shocks and other propagating discontinuities maybe generated but also proves that the curvature flow has long time existence solutions.

The parabolic theory for the evolving of plane curves has been extremely successful in providing geometers with great insight (see Gage and Hamilton [1], Grayson [2] and Zhu [3]). It can be applied to many physical problems such as crystal growth, computer vision and image processing. One of the other important applications of mean curvature flow is that Huisken and Ilmanen developed a theory of weak solutions of the inverse mean curvature flow and used it to prove successfully the Riemannian Penrose inequality which plays an important role in general relativity (see [4]).

However, to our knowledge, there is very few hyperbolic versions of mean curvature flow. In fact, Yau in [5] has suggested the following equation related to a vibrating membrane or the motion of a surface

$$\frac{\partial^2 X}{\partial t^2} = H\vec{n}, \tag{1.3}$$

where H is the mean curvature and $\vec{n}$ is the unit inner normal vector of the surface. Gurtin and Podio-Guidugli [6] developed a hyperbolic theory for the evolution of plane curves. Rotstein, Brandon and Novick-Cohen [7] studied a hyperbolic theory by the mean curvature flow equation

$$v_t + \psi v = k, \tag{1.4}$$

where v_t is the normal acceleration of the interface, ψ is a constant. A crystalline algorithm was developed for the motion of closed polygonal curves. He, Kong and Liu [8] investigated the hyperbolic mean curvature flow (1.3) and gave the short-time existence theorem. Furthermore, the nonlinear stability defined on the Euclidean space with dimension larger than 4 was proved. The hyperbolic mean curvature flow was considered as one of the general hyperbolic geometric flows introduced by Kong and Liu (see [9] for more discussions for related hyperbolic flows and their applications to geometry and Einstein equations). Lefloch and Smoczyk [10] studied the following geometric evolution equation of hyperbolic type which governs the evolution of a hypersurface moving in the direction of its mean curvature vector

$$\begin{cases} \dfrac{\partial^2}{\partial t^2} X = eH(u, t)\vec{N} - \nabla e, \\ X(u, 0) = X_0, \\ \left(\dfrac{\partial X}{\partial t}\right)^{\vec{T}_0}_{t=0} = 0, \end{cases} \tag{1.5}$$

where $\vec{T}_0$ stands for the unit tangential vector of the initial hypersurface X_0,

$$e = \frac{1}{2}\left(\left|\frac{\partial X}{\partial t}\right|^2 + n\right)$$

is the local energy density and $\nabla e \triangleq \nabla^i e_i$, in which $e_i = \frac{\partial e}{\partial x^i}$. This flow stems from a geometrically natural action containing kinetic and internal energy terms. They have shown that the normal hyperbolic mean curvature flow will blow up in finite time. In the case of graphs, they introduce a concept of weak solution suitably restricted by an entropy inequality and proved that the classical solution is unique in the larger class of entropy solutions. In the special case of one-dimensional graphs, a global-in-time existence result is established. Moreover, an existence theorem has been established under the assumption that the BV norm of initial data is small. Subsequently, the hyperbolic curve shortening problem is studied in Kong, Liu and

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