Nonlinear Analysis 97 (2014) 145–154

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Local well-posedness and wave breaking results for periodic solutions of a shallow water equation for waves of moderate amplitude

Nilay Duruk Mutlubaş

Istanbul Kemerburgaz University, Faculty of Arts and Sciences, Mahmutbey Dilmenler Caddesi, No. 26, 34217 Bağcılar, Istanbul, Turkey

ARTICLE INFO

Article history:
Received 10 June 2013
Accepted 20 November 2013
Communicated by Enzo Mitidieri

Keywords:
Quasilinear hyperbolic equation
Well-posedness
Wave breaking

ABSTRACT

We study the local well-posedness of a periodic nonlinear equation for surface waves of moderate amplitude in shallow water. We use an approach proposed by Kato which is based on semigroup theory for quasi-linear equations. We also show that singularities for the model equation can occur only in the form of wave breaking, in particular surging breakers.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

We are concerned with an evolution equation which models the propagation of surface waves of moderate amplitude in the shallow water regime:

$$u_t + u_x + \frac{3}{2}\varepsilon u u_x - \frac{3}{8}\varepsilon^2 u^2 u_x + \frac{3}{16}\varepsilon^3 u^3 u_x + \frac{\mu}{12}(u_{xxx} - u_{xxt}) + \frac{7\varepsilon}{24}\mu(u u_{xxx} + 2u_x u_{xx}) = 0, \quad x \in \mathbb{R},\ t > 0. \tag{1}$$

Here $u(x, t)$ is the free surface elevation and ε and μ represent the amplitude and shallowness parameters, respectively.

Since the exact governing equations for water waves fail to provide explicit solutions, many approximate model equations have been proposed, which are based on linear theory and were therefore inadequate to explain potential nonlinear behaviours like wave breaking or solitary waves. Hence, many competing nonlinear models have been suggested to manage these phenomena. One of the most prominent examples is the Camassa–Holm (CH) equation [1] which is an integrable, infinite-dimensional, Hamiltonian system [2–4]. The Cauchy problem associated to the CH equation is known to be locally well-posed for initial data u_0 in Sobolev space H^s, $s > 3/2$ [5,6] as well as periodic Cauchy problem [7,8]. A significant feature of the CH equation is that some of the bounded classical solutions develop singularities in finite time in the form of wave breaking, i.e. their slope becomes unbounded [9–13]. Beyond the breaking time, the solutions recover in the sense of global weak solutions [14,15]. The relevance of the CH equation as a model for the propagation of shallow water waves was discussed by Johnson [16], where he showed that the CH equation describes the horizontal component of the velocity field at a certain depth within the fluid; see also [17]. Following the ideas presented therein, Constantin and Lannes derived the evolution Eq. (1) for the free surface which approximates the governing equations to the same order as the CH equation [18].

Local well-posedness results for the initial value problem associated to (1) was first proved by Constantin and Lannes [18] for initial data $u_0 \in H^s$, $s > 5/2$. It has been recently shown that local well-posedness can also be obtained for a class of

E-mail address: nilay.duruk@kemerburgaz.edu.tr.

0362-546X/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.na.2013.11.021

initial data comprising less regular data $u_0 \in H^s, s > 3/2$ additionally [19]. The well-posedness in the context of Besov spaces together with the regularity and the persistence properties of strong solutions are studied in [20]. The model equation also possesses solitary travelling wave solutions decaying at infinity [21]. Their orbital stability has been recently studied using an approach proposed by Grillakis, Shatah and Strauss [22], and taking advantage of the Hamiltonian structure of (1) [23].

In the present paper, we look for solutions of the Cauchy problem corresponding to (1) which are spatially periodic of period 1. We conclude local well-posedness by employing an approach due to Kato using semigroup theory for quasi-linear equations. However the solutions depend continuously on their corresponding initial data, it has been recently shown that this dependence is not uniformly continuous [24]. Moreover, we prove that singularities arise in finite time in the form of breaking waves and we provide the sufficient condition on the initial data so that wave breaking occurs.

Before we give the local well-posedness results, we state the theory Kato proposed:

2. Kato's theory

Consider the abstract quasi-linear evolution equation in the Hilbert space X:

$$u_t + A(u)u = f(u), \quad t \geq 0, \qquad u(0) = u_0. \tag{2}$$

Let Y be a second Hilbert space such that Y is continuously and densely injected into X and let $S : Y \to X$ be a topological isomorphism. Assume that

(A1) Given $C > 0$, for every $y \in Y$ with $\|y\|_Y \leq C$, $A(y)$ is quasi-m-accretive on X, i.e. $-A(y)$ is the generator of a C_0 quasi-contractive semigroup $\{T(t)\}_{t\geq 0}$ in X satisfying $\|T(t)\| \leq e^{wt}$.

(A2) For every $y \in Y$, $A(y)$ is a bounded linear operator from Y to X and

$$\|(A(y) - A(z))w\|_X \leq c_1\|y - z\|_X\|w\|_Y, \quad y, z, w \in Y.$$

(A3) $SA(y)S^{-1} = A(y) + B(y)$, where $B(y) \in \mathcal{L}(X)$ is uniformly bounded on $\{y \in Y : \|y\|_Y \leq M\}$. Moreover,

$$\|(B(y) - B(z))w\|_X \leq c_2\|y - z\|_Y\|w\|_X, \quad w \in X.$$

(A4) The function f is bounded on bounded subsets $\{y \in Y : \|y\|_Y \leq M\}$ of Y for each $M > 0$, and is Lipschitz in X and Y:

$$\|f(y) - f(z)\|_X \leq c_3\|y - z\|_X, \quad \forall y, z \in X$$

and

$$\|f(y) - f(z)\|_Y \leq c_4\|y - z\|_Y, \quad \forall y, z \in Y.$$

Here c_1, c_2, c_3 and c_4 are constants depending only on $\max\{\|y\|_Y, \|z\|_Y\}$.

Theorem 2.1. *([25,26]) Assume* (A1), (A2), (A3), (A4) *hold. Given $u_0 \in Y$, there is a maximal $T > 0$, depending on u_0, and a unique solution u to (2) such that*

$$u = (u_0, .) \in C([0, T), Y) \cap C^1([0, T), X).$$

Moreover, the map

$$u_0 \mapsto u(u_0, .) : Y \to C([0, T), Y) \cap C^1([0, T), X)$$

is continuous.

3. Local well-posedness

Consider the following periodic Cauchy problem where (1) is rewritten in quasi-linear equation form:

$$u_t + A(u)u = f(u) \quad x \in \mathbb{R},\ t > 0, \tag{3}$$

$$u(x, 0) = u_0(x) \quad x \in \mathbb{R}, \tag{4}$$

$$u(x, t) = u(x + 1, t) \quad x \in \mathbb{R},\ t > 0, \tag{5}$$

with

$$A(u) = -\left(1 + \frac{7}{2}\varepsilon u\right)\partial_x, \tag{6}$$

$$f(u) = -\left(1 - \frac{\mu}{12}\partial_x^2\right)^{-1}\partial_x\left[2u + \frac{5}{2}\varepsilon u^2 - \frac{1}{8}\varepsilon^2u^3 + \frac{3}{64}\varepsilon^3u^4 - \frac{7}{48}\varepsilon\mu u_x^2\right]. \tag{7}$$

Hereafter, we denote the Sobolev space of functions of period 1 by H^s_{per}.

Download English Version:

https://daneshyari.com/en/article/839954

Download Persian Version:

https://daneshyari.com/article/839954

Daneshyari.com