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Steady states of a predator–prey model with prey-taxis[☆]

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ABSTRACT

We study the steady states of a predator–prey model with prey-taxis incorporating Holling type II functional response under the homogeneous Neumann boundary condition. The stability of equilibrium points and the existence of non-constant steady states are investigated. We obtain that the prey-tactic sensitivity coefficient delays the stability of the unique positive constant solution, but for other equilibrium points' stability, the prey-tactic sensitivity coefficient does not influence on it. Furthermore, we derive some sufficient conditions relative to the prey-tactic sensitivity coefficient which confines the existence of steady states and find that even if the interaction coefficient is sufficiently large, there also exist non-constant positive steady states under some conditions.

1. Introduction

The coexistence of prey population and predator population can be mostly described by the presence of positive steady states and spatial patterns of the populations can be characterized by non-constant steady states, which have been studied for population models with random diffusions by papers [1–9].

In addition to the random diffusion of predators and the prey, the spatial–temporal variations of the predator's velocity are often directed by prey taxis, which is defined as the movement of predators controlled by the prey density. Paper [10] showed that predators in area-restricted search tend to move towards areas with high food abundance to increase the efficiency of foraging. Other studies measuring characteristics of individual movement support the mechanism of prey-taxis [11,12]. The prey-taxis equation was derived by paper [10] and was extended by [13]. Paper [14] studied the following predator–prey model with prey-taxis:

$$\begin{cases} \dfrac{\partial u}{\partial t} - d_1 \triangle u + \nabla \cdot (\alpha u \nabla v) = -au + \beta \dfrac{cuv}{m+bv} & \text{in } \Omega \times (0,T), \\ \dfrac{\partial v}{\partial t} - d_2 \triangle v = r\left(1 - \dfrac{v}{K}\right) v - \dfrac{cuv}{m+bv} & \text{in } \Omega \times (0,T), \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega \times (0,T), \\ (u(0,x), v(0,x)) = (u_0(x), v_0(x)) \geq (0,0) & \text{in } \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in R^N ($N \geq 1$ is an integer) with a smooth boundary $\partial\Omega$; u and v represent the densities of the predator and the prey respectively; positive constants d_1 and d_2 are the random diffusion coefficients of predator

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population and prey population respectively; positive constants $a, K, r, \beta, c/m, b/m$ represent the per capita death rate of predators, the carrying capacity of the prey, the prey intrinsic growth rate, the conversion rate, the searching efficiency, the handling time spent by a predator to catch and consume a prey; α denotes the prey-tactic sensitivity. The term $\alpha u \nabla v$ gives the velocity by which predators move up the gradient of the prey.

In [14], the authors studied the existence of weak solutions for model (1.1) using the Schauder fixed-point theorem, and the uniqueness of the solution via the duality technique. In [15], the author analyzed the classical solution in $C^{2+\sigma,1+\sigma/2} \times C^{2+\sigma,1+\sigma/2}$ $(0 < \sigma < 1)$ to model (1.1). However, there are no studies in its steady states. The main objective of this article is to study the existence and stability of steady-state solutions to model (1.1), which is the continuous work of papers [14, 15]. Motivated by the "volume-filling" mechanism [16,17], we fix

$$\alpha = \alpha(u) =: \begin{cases} \chi \left(1 - \dfrac{u}{u_m}\right) & \text{if } 0 \le u < u_m, \\ 0 & \text{if } u \ge u_m, \end{cases} \tag{1.2}$$

where χ and u_m are positive constants. Moreover, we assume $\partial \Omega \in C^{2+\alpha}$.

In the following discussion, we mainly investigate the case $0 \le u < u_m$, since there have been discussions for the case $u \ge u_m$ [3,4].

This paper is organized as follows. In Section 2, we study the stability of steady states of (1.1). In Section 3, we investigate the existence of periodic solutions from the positive constant solution of (1.1). In Section 4, we analyze the existence of the non-constant steady state of (1.1) by using the Leray–Schauder degree theory.

2. Stability of equilibrium points

In this section we shall study the stability of nonnegative constant steady states of (1.1). Note that a steady state satisfies the following strongly coupled elliptic system:

$$\begin{cases} -d_1 \Delta u + \nabla \cdot (\alpha u \nabla v) = -au + \beta \dfrac{cuv}{m + bv} & \text{in } \Omega, \\ -d_2 \Delta v = rv\left(1 - \dfrac{v}{K}\right) - \dfrac{cuv}{m + bv} & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial \Omega. \end{cases} \tag{2.1}$$

It admits the following three non-negative constant solutions:

(i) the trivial solution $(0, 0)$;
(ii) the semi-trivial solution $(0, K)$;
(iii) the unique positive constant solution $\mathbf{w}_* =: (u_*, v_*)$, where

$$u_* = \frac{\beta m r}{K(\beta c - ab)^2}[K(\beta c - ab) - am], \qquad v_* = \frac{am}{\beta c - ab}.$$

The positive constant solution $\mathbf{w}_*$ exists if and only if

$$a < \frac{\beta c K}{m + bK}, \tag{2.2}$$

and in this case $v_* < K$.

It should be noted that since $m, b, K > 0$, (2.2) implies that $\beta c - ab > \frac{am}{K} > 0$.

Let $0 = \mu_0 < \mu_1 < \cdots < \mu_k < \cdots \to +\infty$ denote the eigenvalues of $-\triangle$ in Ω under the homogeneous Neumann boundary condition and set $\alpha_* = \chi(1 - \frac{u_*}{u_m})$.

Theorem 2.1. *Let (2.2) hold. If*

$$\frac{b}{\beta c} + \frac{m}{\beta c K} < \frac{2m}{K(\beta c - ab)}, \tag{2.3}$$

then $\mathbf{w}_*$ is locally asymptotically stable.

Proof. The linearized problem of (1.1) at $\mathbf{w}_*$ can be expressed by

$$\mathbf{w}_t = (D\Delta + F_{\mathbf{w}}(\mathbf{w}_*))\mathbf{w},$$

where $\mathbf{w} = (u(x, t), v(x, t))^T$,

$$F(\mathbf{w}) = \left(-\chi\left(1 - \frac{2u}{u_m}\right)\nabla u \cdot \nabla v - au + \frac{\beta cuv}{m + bv}, \ r\left(1 - \frac{v}{K}\right)v - \frac{cuv}{m + bv}\right)$$