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ORIGINAL RESEARCH ARTICLE

Stokes transport in layers in the water column based on long-term wind statistics

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Summary This paper addresses the Stokes transport velocity for deep water random waves in given layers in the water column based on wind statistics, which can be estimated by the simple analytical tool provided here. Results are exemplified by using the Phillips and Pierson-Moskowitz model wave spectra together with long-term wind statistics from one location in the northern North Sea and from four locations in the North Atlantic. The results are relevant for e.g. assessing the drift of marine litter in the ocean based on, for example, global wind statistics.

1. Introduction

There has recently been much focus on the environmental issues related to plastic litter in the oceans; see e.g. van Sebille et al. (2015), Sherman and van Sebille (2016), Keswani et al. (2016), Brennecke et al. (2016), Avio et al. (2017); also documenting that plastic litter occurs in different layers in the water column beneath the ocean surface. One important constituent of ocean circulation models is the Stokes drift which contributes to the transport of plastic as well as microplastic located in different layers in the water column. The wave-average of the water particle trajectory in the wave propagation direction gives the Lagrangian velocity referred to as the Stokes drift, while the volume Stokes transport is obtained as the integral over the water depth of the Stokes drift (Rascle et al., 2008). The general background and further details of the Stokes drift are given in e.g. Dean and Dalrymple (1984). Myrhaug et al. (2016) gives a brief review of the literature up to that date (see the references therein). More recent works include those of Breivik et al. (2016), Li et al. (2017) and Myrhaug (2017).

The purpose of the present analytical study is to provide a simple analytical tool which yields estimates of the Stokes transport velocity for deep water random waves in given layers in the water column based on wind statistics. The

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Stokes transport velocity is obtained by integrating the Stokes drift between two elevations in the water column and dividing by the distance between these two elevations. Results are exemplified by using the two model wave spectra by Phillips and Pierson-Moskowitz together with long-term wind statistics from one location in the northern North Sea and from four locations in the North Atlantic. The present results are relevant for estimating the drift of e.g. marine litter in the ocean based on global wind statistics.

2. Theoretical background

Based on classical potential wave theory the time-averaged Lagrangian drift at a z-level in the water column in a water depth d is (Dean and Dalrymple, 1984):

$$u_L^s(z) = \frac{ga^2k^2}{\omega}\frac{\cosh 2k(z+d)}{\sinh 2kd}, \tag{1}$$

where g is the acceleration of gravity, a is the linear wave amplitude, and k is the wave number related to the cyclic wave frequency ω by the dispersion relationship

$$\omega^2 = gk\tanh kd. \tag{2}$$

According to Eq. (1) the drift of the water particles is in the direction of the wave propagation; the maximum is at the mean free surface $z = 0$, and decreases rapidly towards the sea bed as $z \to -d$ (z is positive upwards). This drift velocity is commonly referred to as Stokes drift.

When assessing the transport of material in the water column the drift velocity associated with the Stokes transport in different layers of the water column is a quantity of interest. This drift velocity is obtained by integrating Eq. (1) between two levels $z = -h_2$ and $z = -h_1$ in the water column and divided by the distance between these two levels $\Delta h = h_2 - h_1$, given as

$$v = \frac{1}{\Delta h}\int_{-h_2}^{-h_1} u_L^s(z)dz = \frac{ga^2k}{2\omega} \times \frac{\sinh[2kd(1-h_1/d)]-\sinh[2kd(1-h_2/d)]}{\Delta h\cdot\sinh(2kd)}. \tag{3}$$

In deep water (i.e. for large kd and thus $\omega^2 = gk$ from Eq. (2)) the Stokes transport velocity in Eq. (3) reduces to

$$v = \frac{a^2\omega}{2\Delta h}\left(e^{-2(\omega^2/g)h_1} - e^{-2(\omega^2/g)h_2}\right). \tag{4}$$

If an individual random wave with amplitude a_n and cyclic wave frequency ω_n is considered, then the Stokes transport velocity for individual random waves in deep water is given as

$$v_n = \frac{1}{2}a_n^2\omega_n\frac{1}{\Delta h}\left(e^{-2(\omega_n^2/g)h_1} - e^{-2(\omega_n^2/g)h_2}\right). \tag{5}$$

The wave amplitude is obtained from the wave spectrum $S(\omega)$ as $a_n^2 = 2S(\omega_n)\Delta\omega$ where $\Delta\omega$ is a constant separation between frequencies. By substituting this in Eq. (5) and considering an infinite number of wave components, the total Stokes transport velocity within a sea state of random waves is obtained as

$$V = \frac{1}{\Delta h}\int_0^\infty \omega S(\omega)\left(e^{-2(\omega^2/g)h_1} - e^{-2(\omega^2/g)h_2}\right)d\omega. \tag{6}$$

The two terms in the parenthesis of Eq. (6) represent the attenuation of the wave motion in the water column, which here is approximated by taking ω as the spectral peak frequency ω_p. As a result Eq. (6) becomes

$$V = \frac{1}{\Delta h}\left(e^{-2(\omega_p^2/g)h_1} - e^{-2(\omega_p^2/g)h_2}\right)m_1, \tag{7}$$

where m_1 is the first spectral moment obtained from the definition of the nth spectral moment

$$m_n = \int_0^\infty \omega^n S(\omega)\,d\omega; \quad n = 0, 1, 2, \ldots. \tag{8}$$

By combining Eq. (7) with the spectral mean period $T_1 = 2\pi\, m_0/m_1$, the significant wave height $H_s = 4\sqrt{m_0}$, and that there is a relationship between T_1 and the spectral peak period $T_p = 2\pi/\omega_p$, i.e. $T_1 = \gamma_1 T_p$, Eq. (7) is rearranged to

$$V = \frac{1}{\Delta h}\left(e^{-2(\omega_p^2/g)h_1} - e^{-2(\omega_p^2/g)h_2}\right)\frac{\pi H_s^2}{8\gamma_1 T_p}. \tag{9}$$

Thus, V is defined in terms of the sea state parameters H_s and T_p in deep water.

3. Example of results for two standard wave spectra and long-term wind statistics

Two standard deep water wave spectra are chosen; the Phillips and the Pierson-Moskowitz spectra, which both have been used frequently in contexts discussing the Stokes drift, e.g. see Li et al. (2017) and the references therein. The Phillips spectrum was also used by exemplifying results in Myrhaug et al. (2014, 2016) and in Myrhaug (2017), where the latter reference presented the surface Stokes drift and the Stokes transport using long-term wind statistics from the northern North Sea location used here, and from one location on the northwest Shelf of Australia.

3.1. Phillips and Pierson-Moskowitz spectra

The Phillips spectrum is (Tucker and Pitt, 2001)

$$S(\omega) = \alpha\frac{g^2}{\omega^5}, \quad \omega \geq \omega_p = \frac{g}{U_{10}}, \tag{10}$$

where $\alpha = 0.0081$ is the Phillips constant, and U_{10} is the mean wind speed at the 10 m elevation above the sea surface. By using the definition of the spectral moments in Eq. (8), it follows that

$$H_s = 4\sqrt{m_0} = \frac{2\sqrt{\alpha}}{g}U_{10}^2, \tag{11}$$

$$\gamma_1 = \frac{T_1}{T_p} = \frac{2\pi m_0}{T_p m_1} = \frac{3}{4}, \tag{12}$$

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