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Abstract reaction–diffusion systems with nonlocal initial conditions☆

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ABSTRACT

We prove some sufficient conditions for the existence and global uniform asymptotic stability of C^0-solutions for a class of nonlinear delay reaction–diffusion systems subjected to nonlocal initial conditions. Some applications to a specific reaction–diffusion system are included.

1. Introduction

In this paper we prove an existence and uniform asymptotic stability result for C^0-solutions to the abstract nonlinear delay reaction–diffusion system with nonlocal initial data

$$\begin{cases} u'(t) \in Au(t) + F(t, u_t, v_t), & t \in \mathbb{R}_+, \\ v'(t) \in Bv(t) + G(t, u_t, v_t), & t \in \mathbb{R}_+, \\ u(t) = p(u, v)(t), & t \in [-\tau, 0], \\ v(t) = q(u, v)(t), & t \in [-\tau, 0]. \end{cases} \tag{1.1}$$

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Here $\tau \geq 0$, $A : D(A) \subseteq X \rightsquigarrow X$, $B : D(B) \subseteq Y \rightsquigarrow Y$ are m-dissipative operators, X, Y are Banach spaces, $F : \mathbb{R}_+ \times C([-\tau, 0]; \overline{D(A)}) \times C([-\tau, 0]; \overline{D(B)}) \to X$ and $G : \mathbb{R}_+ \times C([-\tau, 0]; \overline{D(A)}) \times C([-\tau, 0]; \overline{D(B)}) \to Y$ are continuous functions, while $p : C_b([-\tau, +\infty); \overline{D(A)}) \times C_b([-\tau, +\infty); \overline{D(B)}) \to C([-\tau, 0]; \overline{D(A)})$ and $q : C_b([-\tau, +\infty); X) \times C_b([-\tau, +\infty); \overline{D(B)}) \to C([-\tau, 0]; \overline{D(B)})$ are nonexpansive.

Throughout, $C_b(I; X)$ denotes the space of all bounded and continuous functions from the interval I to X, equipped with the sup-norm $\|\cdot\|_{C_b(I;X)}$, while $C_b(I; \overline{D(A)})$ denotes its closed subset consisting of all elements $u \in C_b(I; X)$ satisfying $u(t) \in \overline{D(A)}$ for each $t \in I$. We also denote by $C([a, b]; X)$ the space of all continuous functions from $[a, b]$ to X endowed with the sup-norm $\|\cdot\|_{C([a,b];X)}$ and by $C([a, b]; \overline{D(A)})$ its closed subset containing all $u \in C([a, b]; X)$ with $u(t) \in \overline{D(A)}$ for each $t \in [a, b]$. Finally, as usual, if $u \in C([-\tau, +\infty); X)$ and $t \in [0, +\infty)$, $u_t : [-\tau, 0] \to X$ denotes the continuous delayed function defined by

$$u_t(s) := u(t + s)$$

for each $s \in [-\tau, 0]$. Similarly, one defines $C_b(I; Y)$, $C_b(I; \overline{D(B)})$ and v_t.

For previous results on abstract reaction–diffusion systems subjected to initial or periodic conditions, see: Burlică [1], Burlică and Roşu [2,3], Díaz and Vrabie [4], Necula and Vrabie [5], Roşu [6,7] and the references therein.

Since the pioneering paper of Byszewski [8], there was an increasing interest in the study of evolution equations with nonlocal initial conditions, especially because they represent mathematical models of various phenomena, as those described in Deng [9] and McKibben [10, Section 10.2, pp. 394–398]. As far as non-delay evolution equations are concerned, we mention the papers of Garcia and Reich [11] and Paicu and Vrabie [12]. For delay evolution inclusions with nonlocal initial data, see Vrabie [13,14] and Burlică and Roşu [15].

We emphasize that, even though our result is inspired by both Vrabie [13] and Burlică and Roşu [15], its proof cannot be obtained as a simple application of the above mentioned results. The explanation of this fact comes from our general assumptions which are suitably chosen to handle abstract reaction–diffusion systems but do not fit none of the two cases considered in Vrabie [13] and Burlică and Roşu [15].

The importance of such kind of systems rests in the simple observation that they include reaction–diffusion systems for which one unknown function is subjected to a time-periodic condition while the other one to a mean condition over $[0, +\infty)$, situation which is of great practical interest. See the example in Section 7.

The paper is organized in seven sections. Section 2 contains some preliminaries needed in order to make the presentation self-contained. In Section 3 we state the main result, i.e. Theorem 3.1, while in Section 4 we briefly explain the idea of the proof. Section 5 is mainly devoted to some auxiliary results and in Section 6 we give the complete proof of Theorem 3.1. In Section 7, we analyze an example illustrating the effectiveness of the abstract theory.

2. Preliminaries

The reader is assumed to be acquainted with the basic concepts and results concerning m-dissipative operators and nonlinear evolution equations in Banach spaces, and we refer the reader to Barbu [16] and Vrabie [17] for details. As long as functional differential equations with delay are concerned, we refer to Hale [18]. However, for easy references we begin by recalling some basic concepts and results we will use in the sequel.

Let X be a real Banach space with norm $\|\cdot\|$. Let $x, y \in X$ and $h \in \mathbb{R} \setminus \{0\}$. We denote

$$[x, y]_h := \frac{1}{h}(\|x + hy\| - \|x\|),$$

and we recall that there exists

$$\lim_{h \downarrow 0} [x, y]_h = \inf\{[x, y]_h;\ h > 0\} := [x, y]_+.$$

Remark 2.1. One may easily check out that, for each $x, y \in X$ and $\alpha > 0$, we have

(i) $[\alpha x, y]_+ = [x, y]_+$;
(ii) $|[x, y]_+| \leq \|y\|$.

For other properties of $(x, y) \mapsto [x, y]_+$, see Barbu [16, Proposition 3.7, p. 103].

An operator $A : D(A) \subseteq X \rightsquigarrow X$ is called *dissipative* if for each $x_i \in D(A)$ and $y_i \in Ax_i$, $i = 1, 2$, we have

$$[x_1 - x_2, y_2 - y_1]_+ \geq 0.$$

It is called *m-dissipative* if it is dissipative, and, in addition, $R(I - \lambda A) = X$, for each $\lambda > 0$.

Let $f \in L^1(a, b; X)$ and let us consider the evolution equation

$$u'(t) \in Au(t) + f(t). \tag{2.1}$$