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Weighted pseudo almost automorphic solutions of hyperbolic semilinear integro-differential equations[☆]

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ABSTRACT

In this paper, we deal with weighted pseudo almost automorphic behavior of hyperbolic semilinear integro-differential equations in intermediate Banach spaces, where the nonlinear perturbation is weighted pseudo almost automorphic type or weighted Stepanov-like pseudo almost automorphic type. As applications, some interesting examples are presented to illustrate the main findings.

1. Introduction

In 1962, Bochner [1] introduced the concept of almost automorphy, which is an important generalization of almost periodicity. Since then, this pioneer work attracts more and more attention and is substantially extended in several different directions. Many authors have made important contributions to this theory. For more on almost automorphy and related topics, see the recent books [2–4] and the references therein. Recently, there have been several interesting, natural and powerful generalizations of this notation, such as asymptotic almost automorphy [5], weighted pseudo almost automorphy [6], Stepanov-like pseudo almost automorphy [7], weighted Stepanov-like pseudo almost automorphy [8], and so on. For the historical development of almost automorphy and the relationship between these functions, one refer to [8] for more details. In this paper, we conduct further studies on weighted Stepanov like pseudo almost automorphy, the main idea consists of enlarging the weighted ergodic space, with the help of two weighted functions.

The rapid development of the theory of integro-differential equations has been strongly promoted by the large number of applications in physics, engineering, biology and other subjects. This type of equations have received much attention in recent years and the general asymptotic behavior of solutions is at present an active source of research. Recently, the almost periodicity [9], almost automorphy [10] as well as their various generalizations of integro-differential equations have been extensively explored in the studies [11–15]. Most of the papers obtain the existence and uniqueness of such solutions in Banach space X. On the other hand, let $\alpha \in (0, 1)$, if $A : D(A) \subset X \to X$ is a linear operator (possibly unbounded), and X_α denote an arbitrary abstract intermediate Banach space between $D(A)$ and X. Classical examples of those X_α include the fractional spaces $D((-A)^\alpha)$ [16], the real interpolation spaces $D_A(\alpha, \infty)$ [17] and the Hölder space $D_A(\alpha)$, which coincide

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with the continuous interpolation spaces [18]. Some of the papers consider the existence and uniqueness of solutions in X_α for semilinear differential equations [19–21], and neutral functional-differential equations [22]. However, for the general intermediate Banach space X_α, the existence and uniqueness of weighted pseudo almost automorphic solutions to semilinear integro-differential equations is an untreated topic and this is the main motivation of this paper.

The paper is organized as follows. In Section 2, some notations and preliminary results are presented. Section 3 is divided into two parts. In the first one, Section 3.1, we investigate the existence and uniqueness of weighted pseudo almost automorphic solutions to semilinear integro-differential equations with weighted pseudo almost automorphic coefficients. In the second part, Section 3.2, when the nonlinear perturbation is weighted Stepanov-like pseudo almost automorphic type, we explore the properties of solutions for the same equation. In Section 4, an application to partial differential equation is given.

2. Preliminaries and basic results

Let $(X, \|\cdot\|)$, $(Y, \|\cdot\|_Y)$ be two Banach spaces and $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$ stand for the set of natural numbers, integers, real numbers, and complex numbers, respectively. In order to facilitate the discussion below, we further introduce the following notations:

- $BC(\mathbb{R}, X)$ (resp. $BC(\mathbb{R}\times Y, X)$: the Banach space of bounded continuous functions from $\mathbb{R}$ to X (resp. from $\mathbb{R}\times Y$ to X) with the supremum norm.
- $C(\mathbb{R}, X)$ (resp. $C(\mathbb{R}\times Y, X)$): the set of continuous functions from $\mathbb{R}$ to X (resp. from $\mathbb{R}\times Y$ to X).
- $L(X, Y)$: the Banach space of bounded linear operators from X to Y endowed with the operator topology. In particular, we write $L(X)$ when $X = Y$.
- $L^p(\mathbb{R}, X)$: the space of all classes of equivalence (with respect to the equality almost everywhere on $\mathbb{R}$) of measurable functions $f : \mathbb{R} \to X$ such that $\|f\| \in L^p(\mathbb{R}, \mathbb{R})$.
- $L^p_{loc}(\mathbb{R}, X)$: stand for the space of all classes of equivalence of measurable functions $f : \mathbb{R} \to X$ such that the restriction of f to every bounded subinterval of $\mathbb{R}$ is in $L^p(\mathbb{R}, X)$.

2.1. Sectorial linear operators and the analytic semigroup

Definition 2.1 (*[23]*)**.** A linear operator $A : D(A) \subset X \to X$ is said to be ω-sectorial of angle θ if the following hold: there exist constants $\omega \in \mathbb{R}$, $\theta \in (\pi/2, \pi)$ and $\widetilde{M} > 0$ such that

$$\rho(A) \supset S_{\theta,\omega} := \{\lambda \in \mathbb{C} : \lambda \neq \omega, |\arg(\lambda - \omega)| < \theta\},$$
$$\|R(\lambda, A)\| \le \frac{\widetilde{M}}{|\lambda - \omega|}, \quad \lambda \in S_{\theta,\omega}, \tag{2.1}$$

where $R(\lambda, A) = (\lambda I - A)^{-1}$ for each $\lambda \in S_{\theta,\omega}$.

It is well know that [16] If A is ω-sectorial of angle θ, then it generates an analytic semigroup $(T(t))_{t\ge 0}$ in the sector $S_{\theta-\pi/2,0}$, which maps $(0, \infty)$ to $L(X)$ such that there exist $M_0, M_1 > 0$ with

$$\|T(t)\| \le M_0 e^{\omega t}, \quad t > 0,$$
$$\|t(A - \omega)T(t)\| \le M_1 e^{\omega t}, \quad t > 0.$$

Definition 2.2 (*[23]*)**.** A semigroup $(T(t))_{t\ge 0}$ is said to be hyperbolic, if there exist projection P and constants $M, \delta > 0$ such that each $T(t)$ commutes with P, $Ker\ P$ is invariant with respect to $T(t)$, $T(t) : ImQ \to Im\ Q$ is invertible and

$$\|T(t)Px\| \le Me^{-\delta t}\|x\| \quad \text{for } t \ge 0,$$
$$\|T(t)Qx\| \le Me^{\delta t}\|x\| \quad \text{for } t \le 0,$$

where $Q := I - P$ and $T(t) := (T(-t))^{-1}$ for $t \le 0$.

Recall that if a semigroup $(T(t))_{t\ge 0}$ is analytic, then $(T(t))_{t\ge 0}$ is hyperbolic if and only if

$$\sigma(A) \cap i\mathbb{R} = \emptyset,$$

see for instance [23].

2.2. Intermediate Banach space

Definition 2.3 (*[16]*)**.** Let $\alpha \in (0, 1)$. A Banach space $(X_\alpha, \|\cdot\|_\alpha)$ is said to be an intermediate space between $D(A)$ and X, if $D(A) \subset X_\alpha \subset X$ and there exists a constant $\widetilde{c} > 0$ such that

$$\|x\|_\alpha \le \widetilde{c}\|x\|^{1-\alpha}\|x\|_A^\alpha, \quad x \in D(A),$$

where $\|\cdot\|_A$ is the graph norm of A.