Nonlinear Analysis 95 (2014) 592–606

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Optimal time decay of the Boltzmann system for gas mixtures

Shuangqian Liu *, Hongxia Liu

Department of Mathematics, Jinan University, Guangdong, PR China

ARTICLE INFO

Article history:
Received 13 April 2013
Accepted 29 September 2013
Communicated by Enzo Mitidieri

MSC:
35B20
82B40

Keywords:
Gas mixtures
Kawashima's compensating function
Energy method

ABSTRACT

In this paper, we are concerned with the Boltzmann equation for the mixture of vapors of two gases in the whole space. Given the initial data of one gas near vacuum and the other near a global Maxwellian state, we obtain the global existence and optimal large time behaviors of the Boltzmann system for such binary mixture. The proof is based on Kawashima's compensating function technique and a refined energy method.

1. Introduction

1.1. System for gas mixtures

Consider a binary mixture of hard-sphere gases: gas A and gas B. The Boltzmann equation for such gases takes the form of the following system:

$$\begin{cases} \partial_t F_A + \xi \cdot \nabla_x F_A = Q(F_A, F_A) + Q(F_A, F_B), \\ \partial_t F_B + \xi \cdot \nabla_x F_B = Q(F_B, F_B) + Q(F_B, F_A). \end{cases} \tag{1.1}$$

The right-hand side represents the usual elastic collision terms which for $X, Y \in \{A, B\}$ are given by

$$Q(F_X, F_Y) = \int_{\mathbb{R}^3 \times \mathbb{S}^2_+} |(\xi - \xi_*) \cdot \sigma| \left\{ F'_X F'_{Y_*} - F_X F_{Y_*} \right\} d\xi_* d\sigma.$$

Here $\mathbb{S}^2_+ = \{\sigma \in \mathbb{S}^2 : (\xi - \xi_*) \cdot \sigma \geq 0\}$, $[F_X, F_Y] = [F_X(t, x, \xi), F_Y(t, x, \xi)]$ (use $[\cdot, \cdot]$ to denote the column vector) are non-negative and stand for the number densities of gas A and gas B which have position $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ and velocity $\xi = (\xi_1, \xi_2, \xi_3) \in \mathbb{R}^3$ at time $t \geq 0$. $[F'_X, F'_{Y_*}] = [F_X(t, x, \xi'), F_Y(t, x, \xi'_*)]$; the post-collision velocities ξ' and ξ'_* are given by the formulas

$$\begin{cases} \xi' = \xi + ((\xi_* - \xi) \cdot \sigma)\sigma, \\ \xi'_* = \xi_* - ((\xi_* - \xi) \cdot \sigma)\sigma. \end{cases}$$

* Corresponding author. Tel.: +86 13560483496; fax: +86 2085228237.
E-mail addresses: shqliusx@163.com (S. Liu), hongxia-liu@163.net (H. Liu).

0362-546X/$ – see front matter © 2013 Published by Elsevier Ltd
http://dx.doi.org/10.1016/j.na.2013.09.029

Observe that the mapping (associated with the operator Q) $(\xi, \xi_*) \to (\xi', \xi'_*)$ is an involution which preserves (the mass,) the momentum and the energy

$$\begin{cases} \xi' + \xi'_* = \xi + \xi_*, \\ |\xi'|^2 + |\xi'_*|^2 = |\xi|^2 + |\xi_*|^2. \end{cases}$$

The initial data of the system (1.1) is given by

$$[F_A(0, x, \xi), F_B(0, x, \xi)] = [F_{A,0}(x, \xi), F_{B,0}(x, \xi)].$$

Notice that all the physical parameters, such as the particle masses and the radii of the molecules, and all other involving constants have been chosen to be unit for the simplicity of presentation.

1.2. Reformulation

In the framework of the Boltzmann equation, Takata–Aoki [1] first investigate the condensation–vaporization problem for the mixture of vapors of different species. And many interesting physical problems existing for this important phenomenon, such as the "ghost effect" and "Knudsen layer" for a gas mixture, have been illustrated [2,3].

This paper is concerned with the phenomena related to vapor–vapor mixtures. Our discussion is mainly based on the elementary phenomenon of mass diffusion of a finite amount of gas A dissolved into the surrounding B. From this viewpoint, we write

$$\begin{cases} F_A = f_A\sqrt{\mu}, & \text{for gas with finite total mass amount,} \\ F_B = \mu + f_B\sqrt{\mu}, & \text{for gas in the background,} \end{cases}$$

where the normalized global Maxwellian is defined as

$$\mu = \mu(\xi) = (2\pi)^{-3/2} e^{-|\xi|^2/2}.$$

Then the Cauchy problem (1.1) can be reformulated as

$$\partial_t f_A + \xi \cdot \nabla_x f_A + L_1 f_A = \Gamma(f_A, f_A) + \Gamma(f_A, f_B), \tag{1.2}$$

$$\partial_t f_B + \xi \cdot \nabla_x f_B + L_0 f_B + K_1 f_A = \Gamma(f_B, f_B) + \Gamma(f_B, f_A), \tag{1.3}$$

with initial data

$$[f_A(0, x, \xi), f_B(0, x, \xi)] = [f_{A,0}(x, \xi), f_{B,0}(x, \xi)]. \tag{1.4}$$

Here the linearized collision terms $L_i g$ $(i = 0, 1)$ and the nonlinear collision operator $\Gamma(g, h)$ are respectively defined by

$$\begin{aligned} L_0 g &= -\left\{\mu^{-1/2} Q(\mu^{1/2} g, \mu) + \mu^{-1/2} Q(\mu, \mu^{1/2} g)\right\}, \\ L_1 g &= -\mu^{-1/2} Q(\mu^{1/2} g, \mu), \\ K_1 g &= -\mu^{-1/2} Q(\mu, \mu^{1/2} g), \\ \Gamma(g, h) &= \mu^{-1/2} Q(\mu^{1/2} g, \mu^{1/2} h). \end{aligned} \tag{1.5}$$

1.3. Macro projections, notations and norms

It is easy to see that the null spaces of the linearized operators L_0 and L_1 are given by

$$\mathcal{N}_0 = \text{span}\left\{\mu^{1/2}, \xi_i \mu^{1/2}\ (1 \le i \le 3), |\xi|^2 \mu^{1/2}\right\}$$

and

$$\mathcal{N}_1 = \text{span}\left\{\mu^{1/2}\right\},$$

respectively.

Let $\mathbf{P}_0$ and $\mathbf{P}_1$ be the orthogonal projections from L^2_ξ to $\mathcal{N}_0$ and $\mathcal{N}_1$ respectively. Given $f_X(t, x, \xi)$ $(X \in \{A, B\})$, one can write $\mathbf{P}_0$ and $\mathbf{P}_1$ as

$$\mathbf{P}_0 f_X = a_X(t, x)\mu^{1/2} + \sum_{i=1}^{3} b_{X,i}(t, x)\xi_i \mu^{1/2} + c_X(t, x)(|\xi|^2 - 3)\mu^{1/2},$$

and

$$\mathbf{P}_1 f_X = a_X(t, x)\mu^{1/2},$$

where the coefficient functions are determined by f_X in the way that

$$a_X = \left\langle \mu^{1/2}, f_X \right\rangle, \qquad b_{X,i} = \left\langle \xi_i \mu^{1/2}, f_X \right\rangle, \qquad c_X = \frac{1}{6}\left\langle (|\xi|^2 - 3)\mu^{1/2}, f_X \right\rangle.$$